Math 107, Ledder **EXAM 2: 2/13/03** **Name** _____

Show your work for full credit. **TA** _____

1. Let $f(x) = x^5 + x^3 + x$.

   (a) (4 points) Show that f has an inverse. You do not need to find a formula for the inverse.

   (b) (6 points) Find the derivative of $f^{-1}(x)$ at $x = 3$.

2. (10 points) A bowl of porridge at 190°F is placed in a 70°F room at 8:00. One minute later the porridge has cooled to 170°F. When will the temperature be 110°F? You must state the differential equation and justify the solution formula, but you do not need to derive the formula.

3. (12 points) Two of the definite integrals can be computed by substitution. Compute these. You must report exact answers and show work for full credit.

   (a) $\displaystyle\int_0^1 \frac{x}{1+x^4}\,dx$

   (b) $\displaystyle\int_0^1 \frac{x^2}{1+x^4}\,dx$

   (c) $\displaystyle\int_0^1 \frac{x^3}{1+x^4}\,dx$

4. (8 points) Derive the formula for the derivative of $y = \arctan x$.

5. (10 points) A spherical water tank of radius 50 feet has its bottom 200 feet above the ground. Derive an integral to determine the amount of work needed to fill the tank half full. You may use the fact that water weighs $\rho g = 62.4\text{lbs/ft}^3$ or you may leave the factor ρg in your answer. Full credit requires enough work to justify your result.

6. (10 points) An object of length 2 meters is placed on the x-axis from $x = 0$ to $x = 2$. The linear density of the object is $3 + \dfrac{x^2}{4}$ kg/m. Determine the center of mass of the object.