Math 107-Sec 35X Spring '05 **Exam 2** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(25pts) Determine by definition whether the improper integrals (a, b) converge. Find the value of any convergent integral. **Make sure to show all details.**

(a) $\int_1^\infty \frac{1}{1+x}\,dx$

(b) $\int_1^e \frac{1}{x\sqrt{\ln x}}\,dx$

(c) Use the Comparison Test to determine whether the improper integral converge: $\int_1^\infty \frac{x-1}{x^3+x+1}dx$

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2(25pts) (No calculators are allowed on this problem).

(a) Evaluate the integral by the method of integration by parts $\int x \sin(3x) dx$

(b) Evaluate the integral by completing squares: $\int \frac{1}{2x^2 + 4x + 10} dx$

(c) Use trigonometric substitution to evaluate $\int \frac{x^2}{\sqrt{1 - x^2}}\, dx$.

(c) Find a correct form of partial fraction for $\frac{x^2 + 2x + 2}{(x + 1)(x^2 - 1)^2}$. **Do not solve for the constants.**

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3(25pts) (a) Find the limit of the sequence $\lim_{n\to\infty} \dfrac{\sqrt{n^2+2}}{n-1}$. **Make sure to include sufficient details.**

(b) Use the Squeeze Theorem to find the limit $\lim_{n\to\infty} \dfrac{2+(-1)^n \sin n}{\ln n}$. **Make sure to include sufficient details.**

(c) Determine whether the sequence $a_n = \dfrac{n}{2^n}$ is increasing, decreasing, or neither. **Show your work.**

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4(25pts) (a) Find the sum of $1 + 2 - 6 + 18 - 54 + \cdots (-1)^{999} 2 \cdot 3^{999}$

(b) Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{4}{k(k+2)}$

(c) Determine if the series if converges or diverges $\displaystyle\sum_{k=1}^{\infty} k \sin \frac{k\pi}{2}$

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2 Bonus Points: Isaac Newton had worked in these areas: (a) optics, (b) astronomy, (c) the existence of God, (d) all above. (*... The End*)