Math 107H   Fall 2017   **Exam 1**

Print Your Name Legibly: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   **Score:** ⎯⎯⎯⎯⎯⎯⎯⎯

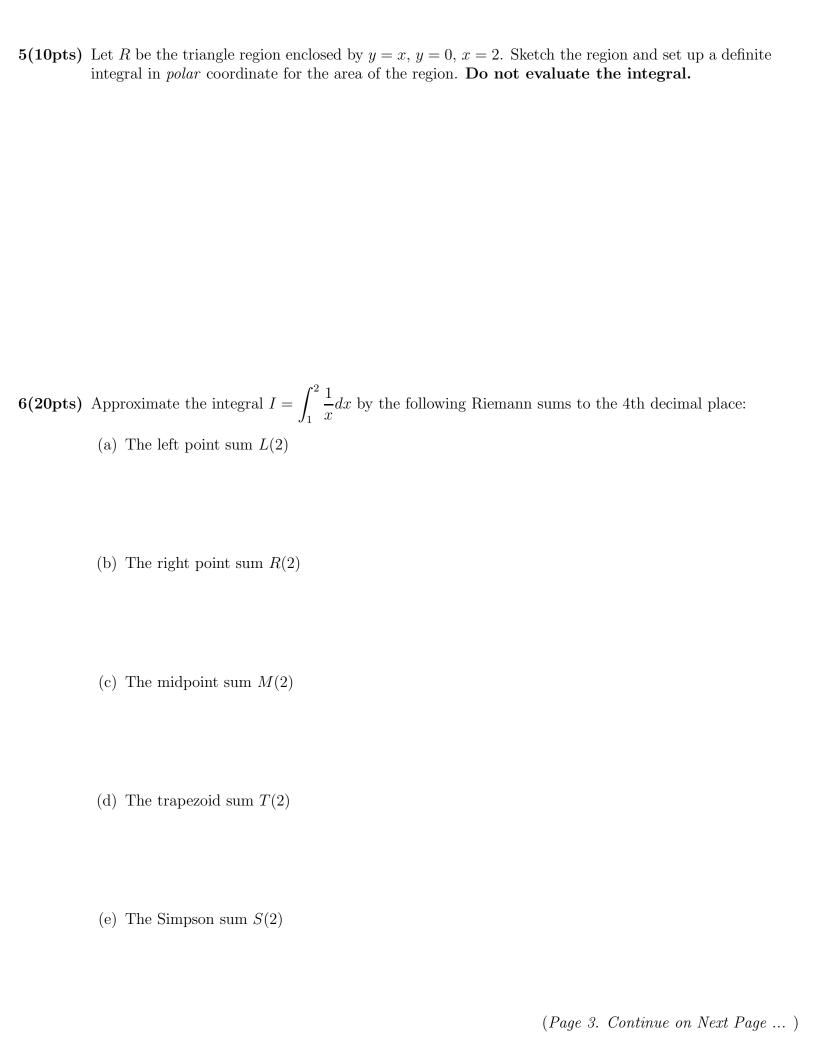**Instructions:** You must show supporting work to receive full and partial credits. No textbook, lecture notes, or formula sheets are allowed.

1(15pts) Use trigonometric substitution to evaluate $\displaystyle\int \frac{\sqrt{x^2 - 1}}{x^2}\, dx$. (You can use the fact that an antiderivative of $\sec x$ is $\ln|\sec x + \tan x|$.)

2(10pts) Use the definition to determine if the improper integral converge: $\displaystyle\int_2^\infty \frac{1}{x \ln x}\, dx$. (**Show work.**)

3(10pts) The base of a solid is the upper half disk bounded by $x^2 + y^2 = 4$, $y \geq 0$. Each cross section of the solid that is perpendicular to the x-axis is an isosceles right triangle, with the right angle touching the x-axis. Set up an integral for the volume of the solid. **Do not evaluate the integral.**

4(15pts) Find the partial fraction for the rational function $\dfrac{x + 4}{(x + 1)(2x^2 + 3x + 4)}$.

5(10pts) Let R be the triangle region enclosed by $y = x$, $y = 0$, $x = 2$. Sketch the region and set up a definite integral in *polar* coordinate for the area of the region. **Do not evaluate the integral.**

6(20pts) Approximate the integral $I = \int_1^2 \frac{1}{x} dx$ by the following Riemann sums to the 4th decimal place:

(a) The left point sum $L(2)$

(b) The right point sum $R(2)$

(c) The midpoint sum $M(2)$

(d) The trapezoid sum $T(2)$

(e) The Simpson sum $S(2)$

7(10pts) Set up an integral for the length of the curve $y = \cos x$ for $0 \leq x \leq \pi$. **Do not evaluate the integral.**

8(10pts) Use the Basic Comparison Test to determine whether the improper integral converges:

$$\int_1^\infty \frac{\sqrt{x} + 5}{x^2 - \sin x + 1} \, dx$$

(**Show work.**)