

Name: \_\_\_\_\_

TA's Name: \_\_\_\_\_

**Instructions:** You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

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**1(15pts)** (a) Find the first three nonzero terms for the Taylor series of the function  $f(x) = \sqrt{x}$  at the point  $x = 1$ .

(b) Use the Taylor series  $\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$  to find the Taylor series of  $x \ln(1-2x)$  at  $x = 0$ .

**2(10pts)** Which two of the three vectors are perpendicular,  $\vec{u} = \langle 1, 2, 3 \rangle$ ,  $\vec{v} = \langle 1, 0, -2 \rangle$ ,  $\vec{w} = \langle 2, -1, 0 \rangle$ ? Show work to support your answer.

**3(15pts)** Use either the Lagrange Error Bound estimate or the Alternating Series Error Bound estimate to determine the number of terms from the Taylor series of  $e^x$  that are needed to approximate the value of  $e^{-0.1}$  to 4 decimal places accuracy.

**4(10pts)** A freshly brewed cup of coffee of  $200^\circ$  F is placed in a  $70^\circ$  F room. Use Newton's Law of Heating and Cooling to write down a differential equation with initial condition for the temperature  $H$  of the coffee as a function of time in minutes. **Do Not Solve The Equations.**

**5(15pts)** Find the solution to the differential equation  $\frac{dy}{dt} = \frac{2t+1}{e^y}$  that also satisfies the initial condition  $y(0) = 1$ .

**6(10pts)** Which is traveling faster, a car whose velocity vector is  $21\vec{i} + 35\vec{j}$ , or a car whose velocity is  $40\vec{i}$ , assuming that units are the same for both directions? Show work to support your answer.

**7(15pts)** Let  $\vec{u} = \langle 1, 2, 3 \rangle$  and  $\vec{v} = \langle -1, 0, 1 \rangle$ .

(a) Find the angle  $\theta$  between the two vectors.

(b) Find the projection,  $\vec{v}_{\text{parallel}}$ , of vector  $\vec{v}$  on vector  $\vec{u}$ .

**8(10pts)** Find an equation for the plane which contains the point  $(1, 0, -2)$  and is perpendicular to the vector  $\vec{n} = \langle 3, 2, 1 \rangle$ .