Dear Calculus Student,

I just got a call from my cousin Janice 'The Juice', whose boss, Frankie 'The Needle' Neidermeier, needs help figuring some odds. He is thinking about setting up a game in his casino called Double Elimination. The gambler plays against the house in a series of rounds. Frankie hasn't decided yet whether the rounds will be cards or dice or what. You keep score and the house wins if it's up by two; the gambler wins if he's up by one. Anyway, Frankie can figure out what the probability is that the house wins one round—it'll be some number p between 0 and 1—and so the probability the gambler wins a round will be $1 - p$. Frankie said that the results of different rounds are independent, so you multiply the probabilities. So from the house's point of view the probability of Win-Loss-Win is $p(1 - p)p$.

Frankie needs to know a bunch of stuff, like what's the probability of the house winning in 2 rounds, 3 rounds, 4 rounds, *any* number of rounds. But what he really wants to know is what's the probability of the house winning a game. The answer needs to be simple. Jimmy says that Frankie said something about adding an infinite number of terns. Where am I gonna get infinitely many seabirds at this time of year?! Could you maybe find the break-even point for the house, and show how the house's probability of winning a game depends on p? Pictures are always good where Frankie's concerned.

Janice has got another problem too. While she was talking to Frankie his head maintenance guy comes in. Hearing that Janice was going to help Frankie with some math he said he had a problem too. He just got the technical specs for these light bulbs that Frankie buys like a billion of a year, and the manual says:

> *To lifetime of bulb density select*
>
> $$p(t) = \begin{cases} cte^{-t^2} & t \geq 0 \\ 0 & t < 0 \end{cases}$$
>
> *Standard integrate for find not changing c.*

and not much else. The maintenance guy needs to know what fraction of these light bulbs will fail in the first six months, between six months and a year, and so on. He figures none of them are going to last much past 5 years. He said he had figured out that the manual was talking about $p(t)$ being a density function for the lifetime of the lightbulbs. He found a reference book that might help; his old copy of Hughes-Hallett had some stuff about this kind of thing in sections 8.7 and 8.8.

Janice really needs some help with this. We need to know the answers, but we also need to convince Frankie. He took a bunch of math, but it was a long time ago, so you've got to explain the answers carefully. I *really* hope you can help. Yours sincerely,

Blaise Pascal
Las Vegas, NV

Projects

Your project report is the solution to an open-ended multistep problem, formally presented. It will probably require several meetings for your group to find a solution to the problem and to present that solution clearly and understandably. Everyone in the group should contribute to the project.

The intent of projects is to expose you to mathematics as you might meet it in the real world, i.e., working as a team. Your group must understand the problem; translate it into mathematics; learn, read about, or develop mathematical methods to find the answer; show that the answer is correct; translate the mathematical answer back into the original problem and, finally, explain the significance of the translated answer. Projects are easier than real world problems, in that we make sure that the problem can be solved using the methods of this course. You may need to learn some new information to do the project.

Preparing formal reports is an important job skill for mathematicians, scientists, and engineers. For example, the Columbia Investigation Board, in its report on the causes of the Columbia space shuttle accident, wrote:

> During its investigation, the board was surprised to receive [PowerPoint] slides from NASA officials in place of technical reports. The board views the endemic use of PowerPoint briefing slides instead of technical papers as an illustration of the problematic methods of technical communication at NASA.

Your report

Your group should write up a short paper (2 and a half to 5 pages would be reasonable) explaining the problem and the mathematics you used to solve it, and then discussing the significance of your solution. Your paper should be a grammatically correct, organized discussion of the problem, with an introduction and a conclusion. While you should answer the specific questions asked in the project, your report should *not* be a disconnected set of answers but a connected narrative with transitions. It should conform to proper English usage (yes, spelling counts) and should include appropriate diagrams and/or graphs, clearly labeled. You should show enough relevant calculations to justify your answers but not so much as to obscure the calculations' purpose. In other words, do not include every calculation, but do include sample calculations. Your report should be typed, but it is fine to leave blank spaces and write the equations in.

Explain your results and conclusions. Assume that your reader is someone who took a calculus class course a while ago and does not remember all of the details. Be sure to avoid plagiarism. The names and recitation numbers of everyone contributing to the report must be listed on the cover page.