Math 107H, Fall '10 **Final Exam** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) Find the following integrals.

(a) $\int x\sqrt{4+x^2}\,dx$

(b) $\int x\sec^2 x\,dx$

2(10pts) Determine if the improper integral converges or diverges: $\int_1^\infty \frac{\sqrt{x}+1}{2x^2-x+1}\,dx$.

(*Continue on Next Page ...*)

3(15pts) The anchor of a boat weight 1500 lb and the chain connected to the anchor weighs 96 lb per foot. Find the minimal amount of work to roll up the anchor together with its chain from the bottom of a harbor 45 feet below the water to the deck of the boat 15 feet above the water.

4(15pts) Find the partial fraction for the rational function $\dfrac{x+1}{x^4+x^3+x^2}$.

(*Continue on Next Page ...*)

5(15pts) The base of a solid is a circle of radius a, and each of its cross section perpendicular to a line through the center of the circle is a square. Set up an integral for the volume of the solid but do not evaluate the integral even if it is simple.

6(15pts) For the following series, either prove it diverges or find its value.

(a) $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{2^{n+2}}{3^{n+3}}$

(b) $\sum_{n=1}^{\infty}\ln(1+\frac{1}{n})$

(*Continue on Next Page ...*)

7(30pts) Determine if the following series absolutely converges, conditionally converges, or diverges.

(a) $\displaystyle\sum_{n=2}^{\infty}(-1)^n\frac{1}{\sqrt{n}\ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\left(1-\frac{1}{n}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty}\frac{n\sin n+1}{2n^{5/2}+\sqrt{n}+10}$

(*Continue on Next Page ...*)

8(15pts) Find the radius of convergence only for the power series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n$.

9(15pts) Write the integral $\int_0^1 \frac{\sin x^2}{x}$ as an infinite series and determine the number of terms needed in order to approximate the integral to the 4th decimal place.

(*Continue on Next Page ...*)

10(10pts) Set up an integral in polar coordinate for the area inside two circles: $r = 1$ and $r = 2\cos\theta$. Do not evaluate the integral. (Suggestion: Sketch the curves and make use of symmetry.)

11(10pts) Find all points on the curve $r = 1 + \cos\theta$ at which the tangent line is horizontal. (No work, no credit.)

(*Continue on Next Page ...*)

12(20pts) (a) Find the angle between the x-axis and the position vector of the point $P(1, 1, 1)$.

(b) Find an equation of the plane containing the x-axis and the point P.

13(10pts) A particle of mass m is accelerated by a force $\mathbf{F} = m\mathbf{a}$ with $\mathbf{a} = -(\mathbf{i}+\mathbf{j}+\mathbf{k})$ from the initial position $\mathbf{r}(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$ at rest. Find its position and velocity one unit time later.

---

2 Bonus Points: Your TA's last name is: _____ (*... The End*)