Math 107-Sec 25X Fall '03 **Exam 1** **Score:** _____

Name: _____ **TA's Name:** _____

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1. (27 points, 9 points each) Evaluate each of the following integrals **(No calculators allowed on this problem).**

   (a) $\int \frac{x}{\sqrt{1+4x^2}} dx$

   (b) $\int \frac{1}{1+4x^2}\, dx$

   (c) $\int_0^5 \frac{x}{\sqrt{4+x}} dx$

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2. (24 points, 8 points each) Let R be the region enclosed by $x = 1 + y^2$, $y = x - 1$.
(a) Find **(but don't evaluate)** an integral whose value gives the exact area of the region R.

(b) Find **(but don't evaluate)** an integral whose value gives the volume of the solid obtained by revolving the region R about the y-axis, **using the method of shells.**

(c) Find **(but don't evaluate)** an integral whose value gives the volume of the solid obtained by revolving the region R about the vertical line $x = 0$, **using the method of cylindrical washers.**

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3. (12 points) A water tank is in the shape of a right circular cone of altitude 10 feet and base radius 5 feet, with its base at the ground. If the tank is full, find an integral **but do not evaluate** for the work done in pumping the top 5 feet of the water out of the top.

4. (10 points, 2 points each) The following table gives some values of a function $y = f(x)$ on the interval $[0, 1]$:

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	0	0.2	0.4	1.0	1.6	2.0

Approximate the value of the integral $\int_0^1 f(x)dx$ by the following Riemann sums:

(a) The left point sum L_3 (Think about how you should treat $f(x)$ beyond the point $x = 1$)

(b) The right point sum R_3

(c) The midpoint sum M_3

(d) The trapezoid sum T_3

(e) The Simpson sum S_3

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5. (12 points) An underwater observatory has a circular hatch of 1 foot radius on its (vertical) wall with the hatch's top 20 feet below the surface. Write down **but do not evaluate** an integral whose value is the force that is required to hold the hatch in place against the water.

6. (15 points, 5 points each) Find the exact values or formula, and calculator values are not allowed.
   (a) Find values of $\sin^{-1}(\sin(\frac{\pi}{6} + k\pi)), k = 1, 2, 3, \ldots$.

   (b) Find the value of $\tan(\sin^{-1}(1/3))$.

   (c) Simplify $\sin(\sec^{-1}(x))$.

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2 Bonus Points: The headquarter of United Nations is at _____. (... *The End*)