Sept. 24, 2002	MATH 107 Sec 251–257 Exam I	Fall Semester, 2002

Name:________________________ Section:______________ Score:________________

Instructions: You must show supporting work to receive full and partial credits.

1(15pts) The following table gives some values of a function $y = f(x)$ on the interval $[1, 2]$:

x	1	1.25	1.5	1.75	2
f(x)	2	1	0	1	2

Approximate the value of the integral $\int_1^2 f(x)dx$ by the following rules:

(a) LEFT(2) and RIGHT(2)

(b) MID(2)

(c) SIMP(2)

(d) If the exact value of $\int_1^2 f(x)dx = 1.25$, what is the error ERROR(2) for the Simpson's rule? and what do you expect for ERROR(6) to be?

2(15pts) Determine whether or not the improper integral $\displaystyle\int_2^4 \frac{2}{(4-2x)^2}dx$ converges. Make sure you show all the works.

(*Continue on Next Page ...*)

3(10pts) Use a comparison test to determine whether the improper integral $\int_1^{\infty} \frac{y+\sqrt{y}}{3y^3+2y^2+y}dy$ converges. State specifically the integral that you compare with and why the given integral converges or diverges.

4(15pts) Evaluate the following integrals. No calculators are allowed.

(a) $\int (t^2+2)^2 dt$

(b) $\int t(t^2+2)^{200} dt$

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5(15pts) Evaluate the integral $\int \frac{x}{x^2+5x+6}dx$, using complete squaring or partial fraction. No calculators allowed.

6(15pts) Evaluate the following integrals, using integration by parts. No calculators allowed.

(a) $\int x \ln x dx$

(b) $\int t \sin t dt$

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7(15pts) Evaluate the following integrals, using substitution. No calculators allowed.

(a) $\int x\sqrt{x+1}dx$

(b) $\int \sin^3\theta d\theta$

2 Bonus Points: Dr. Deng's office hour on Fridays is:__________

The End

Solu. Key Test 1. Math 107

1 (a) $L(2) = (f(1) + f(1.5))(.5) = 1$. $R(2) = (f(1.5) + f(2))(.5) = 1$

(b) $M(2) = (f(1.25) + f(1.75))(.5) = 1$. (c) $S(2) = \dfrac{2M(2) + L(2)}{3} = 1$

(d) $E(2) = 1.25 - S(2) = .25$. $E(6) \doteq \frac{1}{3^4} E(2) = 0.00309$

2 $\displaystyle\int_2^4 \frac{2}{(4-2x)^2}dx = \lim_{a\to 2^+} \int_a^4 \frac{2}{(4-2x)^2}dx = \lim_{a \to 2^+} (4-2x)^{-1}\Big|_a^4$

$\displaystyle = \lim_{a\to 2^+}\left(\frac{1}{4-2(4)} - \frac{1}{4-2a}\right) = +\infty$ (diverges) since $\displaystyle\lim_{a\to 2^+} \frac{1}{4-2a} = -\infty$.

3 $\displaystyle\int_1^\infty \frac{y+\sqrt{y}}{3y^3+2y^2+4}dy$. $\displaystyle\frac{y+\sqrt{y}}{3y^3+2y^2+4} \le \frac{y+y}{3y^3} = \frac{2}{3}\frac{1}{y^2}$. $\displaystyle\int_1^\infty \frac{2}{3}\frac{1}{y^2}dy = \frac{2}{3}\int_1^\infty \frac{1}{y^2}dy$

converges since $p>1$, By comparison test $\displaystyle\int_1^\infty \frac{y+\sqrt{y}}{3y^3+2y^2+4}dy$ (converges)

4 (a) $\displaystyle\int (t^2+2)^2 dt = \int t^4 + 4t^2 + 4\, dt = \frac{1}{5}t^5 + \frac{4}{3}t^3 + 4t + C$

(b) $\displaystyle\int t(t^2+2)^{200} dt \underset{dw = 2t\,dt}{\overset{w = t^2+2}{=\!=}} \frac{1}{2}\int w^{200} dw = \frac{1}{2}\cdot\frac{1}{201} w^{201} + C = \frac{1}{402}(t^2+2)^{201} + C$

5 $\displaystyle\frac{x}{x^2+5x+6} = \frac{x}{(x+2)(x+3)} = \frac{A}{x+2} + \frac{B}{x+3} = \frac{A(x+3)+B(x+2)}{(x+2)(x+3)} \Rightarrow \begin{array}{l} A+B=1 \\ 3A+2B=0 \end{array}$.

$\Rightarrow 3(1-B) + 2B = 0 \Rightarrow 3 - B = 0,\ B = 3,\ A = -2$.

$\displaystyle\int \frac{x}{x^2+5x+6}dx = \int \frac{-2}{x+2}dx + \int\frac{3}{x+3}dx = 3\ln|x+3| - 2\ln|x+2| + C$

6 (a) $\displaystyle\int x\ln x\, dx \underset{\substack{u=\ln x \\ v = \frac{1}{2}x^2}}{=\!=} \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2\cdot\frac{1}{x}dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C$

(b) $\displaystyle\int t\sin t\, dt \underset{\substack{u=t \\ v=-\cos t}}{=\!=} -t\cos t - \int(-\cos t)dt = -t\cos t + \sin t + C$.

7. (a) $\displaystyle\int x\sqrt{x+1}\,dx \underset{\substack{dw = dx \\ x = w-1}}{\overset{w=x+1}{=\!=}} \int (w-1)w^{\frac{1}{2}}dw = \int w^{\frac{3}{2}} - w^{\frac{1}{2}}dw = \frac{1}{\frac{3}{2}+1}w^{\frac{3}{2}+1} - \frac{1}{\frac{1}{2}+1}w^{\frac{1}{2}+1} + C$

$\displaystyle = \frac{2}{5}w^{\frac{5}{2}} - \frac{2}{3}w^{3/2} + C = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{3/2} + C$

(b) $\displaystyle\int \sin^3\theta\, d\theta = \int \sin^2\theta\cdot\sin\theta\, d\theta = \int (1-\cos^2\theta)\sin\theta\, d\theta$

$\displaystyle\underset{dy = -\sin\theta\, d\theta}{\overset{y = \cos\theta}{=\!=}} \int (1-y^2)(-dy) = -\int (1-y^2)dy = y - \frac{1}{3}y^3 + C = \cos\theta - \frac{1}{3}\cos^3\theta + C$

(2:30 - 3:30)