Note: Solutions to Sample Test 2

[#1] (a) Let h be the water level from the bottom, Δh be the infinitesimal thickness of the water slab at height h. The work required to pump that slab of water over the top rim is $62.5 \times 6^2 \times \Delta h \times (2-h)$ in lb×ft. A Riemann sum approximating the total work is $\sum 62.5(36)(2-h)\Delta h$. (b) $\int_0^2 62.5(36)(2-h)dh = \lim_{\Delta h \to 0} \sum 62.5(36)(2-h)\Delta h$. (c) Total force on one side: $\int_0^2 62.5(2-h)(6)dh$, here $(2-h)$ is the depth measuring from the top rim.

[#2] (a) $\sum \rho(x)\Delta x = \sum(5+4\cos(2x))\Delta x$. (b) $\int_0^2 (5+4\cos(2x))dx$.

[#3] $f(x) = \ln(1-3x)$, $f(0) = \ln 1 = 0, f'(0) = \frac{1}{1-3x}(-3)|_{x=0} = -3, f''(0) = (-1)(1-3x)^{-2}(-3)^2|_{x=0} = -9, f'''(0) = (-1)(-2)(1-3x)^{-3}(-3)^3|_{x=0} = -54$. The Taylor series is

$$f(0) + \frac{f'(0)}{x} + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots = -3x - \frac{9}{2}x^2 - 9x^3 - \cdots$$

[#4]

$$f(x) = x^3 e^{-x^2} = \sum_{k=1}^{\infty} x^3 \frac{(-x^2)^k}{k!} = \sum_{k=1}^{\infty} (-1)^k \frac{x^{2k+3}}{k!} = x^3 - x^5 + \frac{x^7}{2!} + \cdots$$

[#5] $C_n = \frac{n}{4^n}$ $\lim_{n\to\infty} \frac{|C_n|}{|C_{n+1}|} = \lim_{n\to\infty} \frac{n}{n+1} 4 = 4 = R$. The power series converges for $|x| < 4$.

[#6] It is geometric series from the second term onwards, with $3(\frac{2}{5})$ as the leading term and $r = -\frac{2}{5}$ as the ratio. Hence

$$20 + 3\left(\frac{2}{5}\right) - 3\left(\frac{2}{5}\right)^2 + 3\left(\frac{2}{5}\right)^3 - \cdots = 20 + \frac{3(2/5)}{1-(-2/5)} = \frac{146}{7}.$$

[#7] $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{3^k k^2}{k!} x^{4k+1}$. (a) $f^{(101)}(0)$ is the coefficient of the x^{101} times 101!. Since the x^{101} term corresponds to $k = 25$, we have $f^{(101)}(0) = (-1)^{25} \frac{3^{25}(25)^2}{25!}(101!) = -\frac{3^{25}(25)^2}{25!}(101!)$ There is no even terms in the power series, or in other words the coefficients of all even terms are zeros. Therefore $f^{(102)}(0) = 0$. (b) $f'(x) = \sum_{k=0}^{\infty} (-1)^k \frac{3^k k^2 (4k+1)}{k!} x^{4k}$