**Note:** Solutions to Sample Test 2

**[#1]** (a) Let $h$ be the water level from the bottom, $\Delta h$ be the infinitesimal thickness of the water slab at height $h$. The work required to pump that slab of water over the top rim is $62.5 \times 6^2 \times \Delta h \times (2-h)$ in lb×ft. A Riemann sum approximating the total work is $\sum 62.5(36)(2-h)\Delta h$. (b) $\int_0^2 62.5(36)(2-h)dh = \lim_{\Delta h \to 0} \sum 62.5(36)(2-h)\Delta h$. (c) Total force on one side: $\int_0^2 62.5(2-h)(6)dh$, here $(2-h)$ is the depth measuring from the top rim.

**[#2]** (a) $\sum \rho(x)\Delta x = \sum(5+4\cos(2x))\Delta x$. (b) $\int_0^2 (5+4\cos(2x))dx$.

**[#3]** $f(x) = \ln(1-3x)$, $f(0) = \ln 1 = 0, f'(0) = \frac{1}{1-3x}(-3)|_{x=0} = -3, f''(0) = (-1)(1-3x)^{-2}(-3)^2|_{x=0} = -9, f'''(0) = (-1)(-2)(1-3x)^{-3}(-3)^3|_{x=0} = -54$. The Taylor series is

$$f(0) + \frac{f'(0)}{x} + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots = -3x - \frac{9}{2}x^2 - 9x^3 - \cdots$$

**[#4]**

$$f(x) = x^3 e^{-x^2} = \sum_{k=1}^{\infty} x^3 \frac{(-x^2)^k}{k!} = \sum_{k=1}^{\infty} (-1)^k \frac{x^{2k+3}}{k!} = x^3 - x^5 + \frac{x^7}{2!} + \cdots$$

**[#5]** $C_n = \frac{n}{4^n}$ $\lim_{n\to\infty} \frac{|C_n|}{|C_{n+1}|} = \lim_{n\to\infty} \frac{n}{n+1} 4 = 4 = R$. The power series converges for $|x| < 4$.

**[#6]** It is geometric series from the second term onwards, with $3(\frac{2}{5})$ as the leading term and $r = -\frac{2}{5}$ as the ratio. Hence

$$20 + 3\left(\frac{2}{5}\right) - 3\left(\frac{2}{5}\right)^2 + 3\left(\frac{2}{5}\right)^3 - \cdots = 20 + \frac{3(2/5)}{1-(-2/5)} = \frac{146}{7}.$$

**[#7]** $f(x) = \sum_{k=0}^{\infty} (-1)^k \frac{3^k k^2}{k!} x^{4k+1}$. (a) $f^{(101)}(0)$ is the coefficient of the $x^{101}$ times 101!. Since the $x^{101}$ term corresponds to $k = 25$, we have $f^{(101)}(0) = (-1)^{25} \frac{3^{25}(25)^2}{25!}(101!) = -\frac{3^{25}(25)^2}{25!}(101!)$ There is no even terms in the power series, or in other words the coefficients of all even terms are zeros. Therefore $f^{(102)}(0) = 0$. (b) $f'(x) = \sum_{k=0}^{\infty} (-1)^k \frac{3^k k^2 (4k+1)}{k!} x^{4k}$