Math 106-Sec 51X Summer '05 **Exam 3** **Score:** __________

Name: ________________________ **TA's Name:** ____________________

Problem	1	2	3	4	5	6	Total
Score							

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(20pts) Find the following limits. (Make sure to check the conditions of L'Hopital Rule to use it.)

(a) $\lim_{x \to 1} \frac{\sin(\pi x)}{e^{x-1} - 1}$

(b) $\lim_{x \to \infty} x^2(\cos(1/x) - 1)$

(c)(5pts) $\lim_{x \to 0^+} \frac{x^2 + 1}{\ln x}$

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2(20pts) (a) Find the linear approximation for the function $f(x) = \sqrt{3+x}$ at $x = 1$.

(b) Some values of a function $f(x)$ is given below:

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	0.3	0.26	0.22	0.20	0.21

Use its linear approximation to estimate the value of $f(0.11)$.

3(15pts) It is given that a function f is defined for all x and $f'(x) = \dfrac{x^2+x-2}{x^{1/3}}$.

(a) Find all critical points of f.

(b) Find the intervals where f is increasing and decreasing, and determine the local maxima and local minima of f.

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4(15pts) It is given that $f(1) = 0,\ f'(3) = f'(-3) = 0$, and $f'' = 18/x^3$.

(a) Use the second derivative test to determine the local maximum and local minimum of the function f.

(b) Find the intervals where f is concave down and concave down, and the inflection points of f.

5(15pts) Sketch the graph of a continuous function f satisfying the following properties: $f(0) = 1$; $f'(-1) = f'(1) = 0$ and $f'(2)$ does not exist; $f'(x) > 0$ for $x < -1,\ 1 < x < 2,\ 2 < x$; $f'(x) < 0$ for $-1 < x < 1,\ x > 2$; $f''(0) = 0$; $f''(x) < 0$ for $x < 0,\ x > 2$; $f''(x) > 0$ for $0 < x < 2$; and $\lim_{x\to+\infty} f(x) = 3$.

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6(15pts) Sketch the graph of function $f(x) = \dfrac{x-1}{x^2}$, showing all significant features: coordinate intercepts, asymptotes, local extrema, inflection points, intervals of monotonicity and concavity. (Show all works. Calculator generated graph receives no credits.)

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2 Bonus Points: Fermat's Last Theorem was proved by : (a) himself (b) his son (c) Isaac Newton (d) Albert Einstein (e) Andrew Webber (f) Andrew Wiles (... *The End*)