

Name: _____

TA's Name: _____

Instructions: You must show supporting work as much as possible to receive full and partial credits.
No text book, notes, formula sheets allowed.

1(16pts) Find the following limits. (Calculator read-off answers receive no credit.)

(a) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3}$

(a) $\lim_{x \rightarrow 1} \frac{\ln x}{x - 1}$

2(18pts) Let $y = f(x) = x^5 + 37x$.

(a) Find an equation for the tangent line of the function $y = f(x)$ at $x = 1$.

(b) Find the value of dy when $x = 1$ and $\Delta x = 0.2$.

(c) It is given that for a function $y = g(x)$, $g(3) = 1$, $g'(3) = 0.2$. Approximate the value $g(3.1)$ and justify your answer.

(Page 2 and Continue on to Next Page ...)

3(16pts) Water is flowing at the rate of $50 \text{ m}^3/\text{min}$ from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. How fast is the water level falling when the water is 5 m deep?

(Page 3 and Continue on to Next Page ...)

4(16pts) Find the absolute extrema of the function $y = f(x) = x^{2/3}(3 - x)$ in the interval $[-1, 1]$. (Calculator read-off answers receive no credit.)

(Page 4 and Continue on to Next Page ...)

- 5(16pts)** A rectangle plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

6(18pts) The first and second derivatives of the function $y = x^2e^{1/x}$ are given as

$$y' = (2x - 1)e^{1/x}, \quad y'' = \left(\frac{2x^2 - 2x + 1}{x^2} \right) e^{1/x}$$

Sketch the graph of the function, including these information: intercept points, horizontal and vertical asymptotes, extrema and intervals of monotonicity, inflection points and intervals of concavity. (Calculator read-off answers receive no credit.)