NAME:_____

TA's name:_____

| Problem | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Value | | | | | | | | | | | | | | | | 200 |
| Score | | | | | | | | | | | | | | | | |

This exam should have **??** pages; please check that it does. Show all work that you want considered for grading. Calculators are allowed, but **an answer will only be counted if it is supported by all the work necessary to get that answer.** Simplify as much as possible, except as noted: for example, don't write $\cos(\pi/4)$ when you can write $\sqrt{2}/2$. Also, **give exact answers only**, except as noted; for example, don't write 3.1415 for $\pi$. No cheating.

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1. (12 points) For $f(x) = 3x - x^2$, find:
   (a) $\dfrac{f(x+h) - f(x)}{h}$
   (b) $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

2. (14 points) Let $f(x) = 3x^4 - 16x^3 + 24x^2 - 3$.
   (a) Find all critical numbers of $f(x)$.
   (b) Find the absolute maximum and minimum values of $f(x)$ on the interval $[-1, 1]$.

3. (16 points) Suppose $f(x) = \sin x \cos x$.
   (a) Find the equation of the tangent line to the curve $y = f(x)$ at $(\pi/6, \sqrt{3}/4)$.
   (b) Find the local linear approximation to $f(x)$ at $x = \pi/6$.
   (c) Use part (b) to approximate $f(.1 + \pi/6)$. You do not need to simplify your answer.

4. (10 points) Let $C$ be the curve $y = \sin(x) - x/2$. Find a point on $C$ where the slope of the tangent line is 0. Make sure you give both the $x$ and $y$ coordinates of this point.

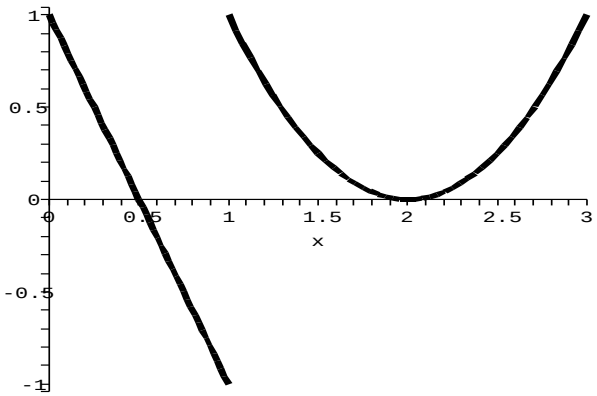
5. (21 points)
   (a) Compute $\lim\limits_{x \to 4} \dfrac{x-4}{x^2 - x - 12}$. (Do not use L'Hopital's Rule.)
   (b) Compute $\lim\limits_{x \to -\infty} \dfrac{x}{\sqrt{2x^2 + 4}}$. (Do not use L'Hopital's Rule.)
   (c) Use L'Hopital's Rule to compute $\lim\limits_{x \to 0} \dfrac{\sin(3x)}{e^{2x} - 1}$. Make sure to verify that L'Hopital's Rule applies.

6. (14 points) Let $s(t) = \dfrac{1}{2}t^3 - 4\sqrt{t}$ be the position of an object at time $t$, where $s$ is measured in feet and $t$ is measured in seconds.
   (a) Find the velocity of the object at time $t = 4$. Make sure that you give the correct units for the velocity.

(b) Find the acceleration of the object at time $t = 4$. Make sure that you give the correct units for the acceleration.

7. (15 points) A closed rectangular box with a square base is to be constructed so that its volume is 32 cubic feet. The material for the sides cost \$2 per square foot while the material for the top and bottom cost \$1 per square foot. Find the dimensions of the box which will minimize the cost.

8. (16 points) The derivative $f'(x)$ of a function $f(x)$ on the interval $[0, 3]$ is shown below.




(a) Determine the interval(s) on which $f(x)$ is increasing and, and the interval(s) on which $f$ is decreasing.

(b) Find all of the critical numbers in $[0, 3]$ and classify each as corresponding to a local minimum, local maximum or saddle point.

(c) In the space next to the given graph, sketch a plausible graph of $y = f(x)$, including the information in parts (a) and (b).

9. (12 points) Suppose a curve $C$ is defined by the equation $\ln y + xy = 0$.

(a) Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.

(b) Find the equation of the tangent line to $C$ at the point $(0, 1)$.

10. (10 points) Suppose $g(x) = f(x^2)$, where we know that $f'(9) = 5$. Find $g'(3)$.

11. (16 points) A jet is taking off on its runway. During part of the takeoff, it has constant acceleration and accelerates from 120 to 240 miles per hour in 1/50 hour. Find the acceleration, and determine how far it travels in that 1/50 hour.

12. (10 points) Let $f(x) = 3x^2 + \dfrac{2}{\sqrt{x}}$.

(a) Find $\displaystyle\int f(x)\,dx$.

(b) Using the Fundamental Theorem of Calculus, find the **exact value** of $\displaystyle\int_0^9 f(x)\,dx$.

13. (8 points) Find $\displaystyle\int \frac{e^{2x}}{e^{2x} + 3}$.

14. (12 points) Sketch a graph of a function that satisfies each of the following conditions: $f'(-1) = 0$; $f'(2) = 0$; $f'(x) < 0$ for $-\infty < x < -1$; $f'(x) > 0$ for $-1 < x < 2$ and $2 < x < \infty$; $f''(x) > 0$ for $-\infty < x < 1$ and $2 < x < \infty$; and $f''(x) < 0$ for $1 < x < 2$.

15. (14 points) Let $f(x) = 4 - x^2$ for $0 \le x \le 2$.

    (a) Draw a picture that represents the Riemann sum with right-endpoint evaluation and $n = 4$ rectangles for $\int_0^2 f(x)\,dx$. Is this Riemann sum an over-estimate or under-estimate of the actual value of the integral?

    (b) Find the value of the Riemann sum in part (a). You do not need to simplify.

    (c) Find the value of the Riemann sum with midpoint evaluation and $n = 4$ for $\int_0^2 f(x)\,dx$. You do not need to simplify.