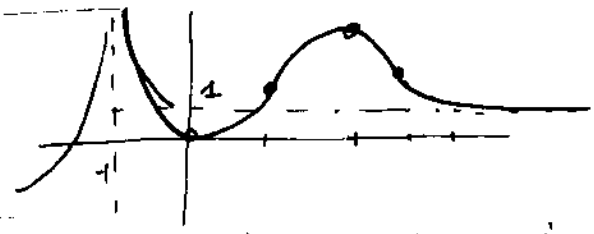
Mat106, Test 3, Soln Key Spring '04

1.



2. $\Delta x = \frac{1-0}{n} = \frac{1}{n}$, $x_i = i\Delta x = \frac{i}{n}$, $R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} \frac{i}{n}\frac{1}{n} = \frac{1}{n^2}\sum_{i=1}^{n} i$

$= \frac{1}{n^2}\frac{n(n+1)}{2} = \frac{1}{2}(1+\frac{1}{n}) \to \frac{1}{2}$ as $n\to\infty$. $\Rightarrow \int_0^1 x\,dx = \lim_{n\to\infty} R_n = \frac{1}{2}$

3. (a) $\lim_{x\to 1} \frac{e^x - e}{x^2-1} = \lim_{x\to 1} \frac{e^x}{2x} = \frac{e}{2} \neq \infty$. $\lim_{x\to -1^+} \frac{e^x-e}{x^2-1} = +\infty$, $\lim_{x\to -1^-} \frac{e^x-e}{x^2-1} = -\infty$

$\Rightarrow$ vertical asymptotes $x=-1$.

(b) $\lim_{x\to\pm\infty} \frac{2x^2+1}{x^2+2x+2} = \lim_{x\to\pm\infty} \frac{2+\frac{1}{x^2}}{1+\frac{2}{x}+\frac{2}{x^2}} = \frac{2+0}{1+0+0} = 2$. H. asypt. $y=2$ on both $x\to\pm\infty$.

4. (a) $\int_0^{50} G(t)\,dt$ = total amount of gas for the trip, in gallons.

(b) $\frac{1}{50}\int_0^{50} G(t)\,dt$ = average rate in gallon per minute.

5 (a) $\int 4x^{10.8} + \frac{2}{\sqrt{x}}\,dx = \frac{4}{11.8}x^{11.8} + \frac{2}{\frac{1}{2}+1}x^{3/2} + C$

(b) $\int \frac{x^2 + x\sin x}{x}\,dx = \int x + \sin x\,dx = \frac{1}{2}x^2 + (-\cos x) + C$

(c) $\int (2x+1)^2\,dx = \int 4x^2 + 4x + 1\,dx = \frac{4}{3}x^3 + 2x^2 + x + C$

(d) $\int F'(x)\,dx = \frac{\sin x^2}{x^2} + C$

6 (a) $\frac{d}{dx}\int \tan e^x\,dx = \tan e^x$ (b) $\int_2^3 f(x)\,dx = \int_1^3 f(x)\,dx - \int_1^2 f(x)\,dx = 3 - 2 = 1$

7. $R_4 = (0 - 0.38 - .4 - .4).5 = -.59$. $L_4 = (.4 + 0 - .38 - .4).5 = -0.19$

$M_4 = (.1 - .12 - .38 - .4).5 = -0.4$. $T_4 = (R_4 + L_4)/2 = -.39$, $S_4 = \frac{1}{3}T_4 + \frac{2}{3}M_4 = -.3966$

8. $Vol = 8\pi = \pi r^2 h$, $Cost = \$2(2\pi r)h + \$1(2)(\pi r^2) \Rightarrow h = \frac{8}{r^2}$

$\Rightarrow$ Min $f(r) = cost = \frac{32\pi}{r} + 2\pi r^2$, $f' = -\frac{32\pi}{r^2} + 4\pi r = \frac{-32\pi + 4\pi r^3}{r^2} = 0$

$\Rightarrow r^3 = \frac{32\pi}{4\pi} = 8 \Rightarrow r = 2$ feet, $h = \frac{8}{r^2} = 2$ feet

9. (a) $\int_0^2 f(x)\,dx = \frac{1}{2}(.5)(1) - \frac{1}{2}(1.5)(1) = -0.5$

(b) $\int_2^3 f(x)\,dx = 0$

Bonus (b) A-