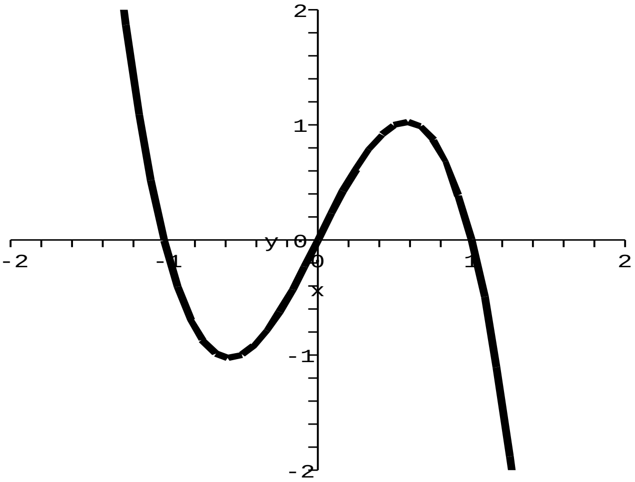
Math 106-Sec 25X Spring '04 **Exam 1** **Score:** __________

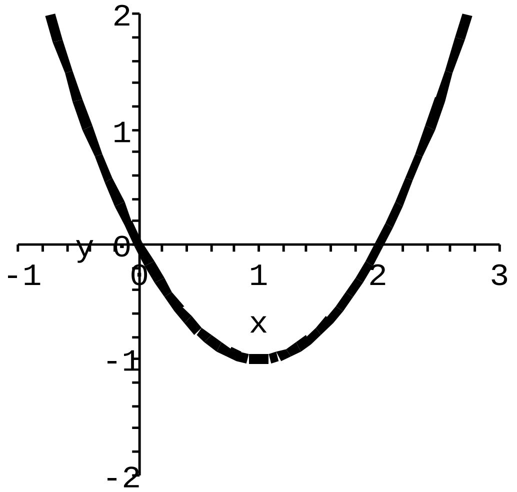
Name: ________________________ **TA's Name:** ____________________

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

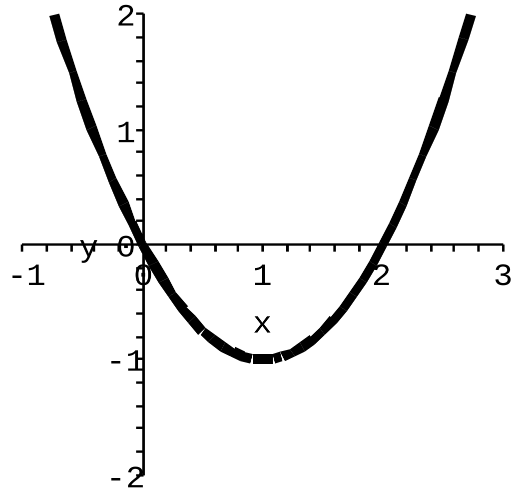
1. (10 pts) The graph of a function $f(x)$ is given below. Superimpose the graphs of (a) $f(2x)$ and (b) $f(x)+1$.



2. (14 pts) (a) The graph of a function $f(x)$ is given, superimpose a plausible graph of $f'(x)$ on the same plot. (b) The graph of the derivative $g'(x)$ is given, superimpose a plausible graph of $g(x)$ satisfying that $g(0)=0$.



(a)



(b)

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3. (14 pts, 7 pts each) Use exact values, that is, if e is the quantity, leave as it is. Answers such as 2.71828... for e are not acceptable and will not receive any credit. So do not use your calculator. Show all of your work.

   (a) Find all values of x that satisfy $\ln x - \ln(x-1) = 1$.

   (b) Let $-\frac{\pi}{2} < \theta < 0$ and $\cos\theta = \frac{1}{3}$. Find the exact value of $\tan\theta$.

4. (14 pts) Using the definition only to find the exact value of $f'(1)$ if $f(x) = (x+1)^2$. Any other method to compute $f'(1)$ will not receive any credit.

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5. (14 pts) Let $f(x) = \dfrac{3x^3 + 2x^2 + x}{x^3 - 1}$.

(a) Evaluate $\lim_{x\to+\infty} f(x)$, and determine whether or not f has a horizontal asymptote. If yes, how many, and what are they?

(b) Does f have any vertical asymptote? If yes, what are they? Explain your answer carefully.

6. (10 pts) Find $f'(x)$ if $f(x) = x^2 + \dfrac{4}{x^2}$. Then find the equation of the tangent line to f at $x = 1$. You can use any method of your choice to compute $f'(x)$.

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7. (14 pts) Let

$$f(x) = \begin{cases} x + 2, x < 1 \\ 2x + k, x \geq 1 \end{cases}$$

where k is a constant.

(a) Find $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$.

(b) Find the value of k for which the function f is continuous on $(-\infty, \infty)$. Make sure to show all of your work to receive full credit.

8. (10 pts) A function f is given in the table below. Estimate $f'(0.4)$ using (a) the forward difference scheme, (b) the backward difference scheme, (c) the average of both.

x	0	0.2	0.4	0.6	0.8	1.0
$f(x)$	0	0.2	0.4	1.0	1.6	2.0

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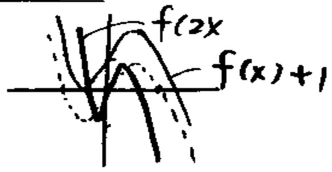
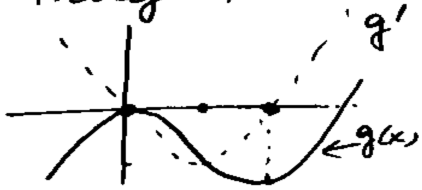
2 Bonus Points: Gottfried Leibniz is a (a) mathematician, (b) a philosopher, (c) a German. (Circle all that are true.)

(... *The End*)

math 106 Exam 1 Soln Key Spring '04

1. 

2 (a) 

(b) 

3. (a) $\ln x - \ln(x-1) = 1 \to \ln\frac{x}{x-1} = 1$, $\frac{x}{x-1} = e^{\ln\frac{x}{x-1}} = e^1 = e$

$x = e(x-1) \Rightarrow (e-1)x = e$, $x = \frac{e}{e-1}$

(b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\sin\theta < 0$ for $-\frac{\pi}{2} < \theta < 0$, $\sin\theta = -\sqrt{1-\cos^2\theta} = -\sqrt{1-\frac{1}{3^2}}$ $= -\frac{2\sqrt{2}}{3}$

$\Rightarrow \tan\theta = \frac{-\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = -2\sqrt{2}$

4. $f'(1) = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{(1+h+1)^2-(1+1)^2}{h} = \lim_{h\to 0}\frac{4+4h+h^2-4}{h}$

$= \lim_{h\to 0}(4+h) = 4$

5. (a) $f(x) = \frac{3x^3+2x^2+x}{x^3-1}$, $\lim_{x\to+\infty} f(x) = \lim_{x\to\infty}\frac{3+\frac{2}{x}+\frac{1}{x^2}}{1-\frac{1}{x^3}} = 3$, similarly

$\lim_{x\to-\infty} f(x) = 3$. horizontal asymptote : $y=3$ in both direction of x

(b) $\lim_{x\to 1^+} f(x) = +\infty$, $\lim_{x\to 1^-} f(x) = -\infty$. $\Rightarrow$ $x=1$ vertical asymptote

6. $f(x) = x^2 + \frac{4}{x^2} = x^2 + 4x^{-2}$, $f'(x) = 2x - 8x^{-3}$, $f'(1) = 2(1) - 8(1)^{-3} = -6$

$f(1) = 1^2 + 4(1)^{-2} = 5$. $\Rightarrow$ tangent line $y-5 = -6(x-1) \Rightarrow y = -6x+11$

7. (a) $\lim_{x\to 1^-} f(x) = \lim_{x\to 1}(x+2) = 3$. $\lim_{x\to 1^+} f(x) = \lim_{x\to 1}(2x+k) = 2+k$.

(b) For $\lim_{x\to 1} f(x) = f(1)$ we need $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) \Rightarrow 3 = 2+k$, $\Rightarrow k=1$

8. (a) $f'(0.4) \doteq \frac{f(0.6)-f(0.4)}{0.2} = \frac{1.0-.4}{.2} = \frac{.6}{.2} = 3$

(b) $f'(0.4) \doteq \frac{f(0.2)-f(0.4)}{0.2-0.4} = \frac{0.2-0.4}{.2-0.4} = 1$

(c) $f'(0.4) \doteq \frac{3+1}{2} = 2$

Bonus : all (a), (b), (c).