April 25, 2002	**MATH 106 Sec 351–352 Exam III**	**Spring Semester, 2002**

Name:______________ Section:__________ Score:__________

Instructions: You must show supporting work to receive full and partial credits.

1(20pts) Consider the function

$$f(x) = x^3 - \ln x, \quad x \geq 0.$$

(a) Find the exact values of all critical points of f. (Calculator estimates will not be accepted.)

(b) Use the 1st Derivative Test to determined if the critical points are local maximum or local minimum.

(c) Use the 2nd Derivative Test to confirm your answer in part (b).

(d) Find the global maximum and global minimum of f in the interval of $(0, 1]$.

(*Continue on Next Page ...*)

2(20pts) (a) Verify that point $(8, 27)$ satisfies the equation $x^{1/3} + y^{1/3} = y - 3x + 2$.

(b) Assume y can be solved as a function of x from the equation above. Find $\dfrac{dy}{dx}$.

(c) Evaluate $\dfrac{dy}{dx}$ at $x = 8$.

(d) Sketch the tangent line of the function $y(x)$ above at $x = 8$.

(*Continue on Next Page ...*)

3(25pts) (a) Use the L'Hopital Rule to find the limits

i. $\lim_{x\to\pi} \dfrac{x\sin x}{x-\pi}$

ii. $\lim_{x\to\infty} xe^{-2x}$

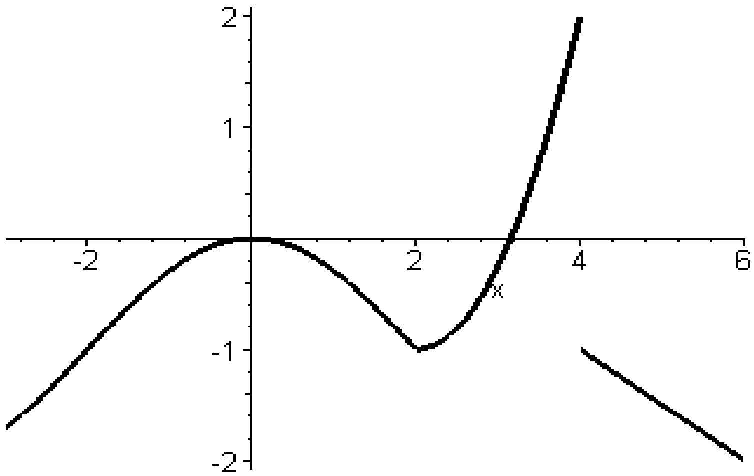
(b) Find the indefinite integrals

i. $\displaystyle\int \frac{x^2+2x}{x}dx$

ii. $\displaystyle\int (\sin x+\cos x)^2 dx$ (*Hint*: $(\sin^2 x)' = 2\sin x\cos x$.)

(*Continue on Next Page ...*)

4(15pts) The derivative f' of a continuous function is given in the graph. Sketch a possible graph for the antiderivative f with $f(0) = 1$ and superpose it in the same graphs as f'. Make sure to list points: (i) Local maxima; (ii) Local minima; (iii) Inflection points



5(20pts) A cylindrical can of 10 cm^3 is made of materials for which the wall and bottom cost 2 cent per cm^2 and the lid costs 4 cent per cm^2. Find the dimensions that minimize the cost.

2 Bonus Points: Your TA's first name is:__________

The End

Spring 02 Solution Key to Test 3 Math 106 Sec 351-353

1 (20pts) (a) $f(x) = x^3 - \ln x$, $f' = 3x^2 - \frac{1}{x} = \frac{3x^3-1}{x} = 0$ $x = \frac{1}{\sqrt[3]{3}}$

(b)

x	$(0, 1/\sqrt[3]{3})$	$(1/\sqrt[3]{3}, +\infty)$
f'	<0	>0

$\Rightarrow f(1/\sqrt[3]{3})$ local minimum.

(c) $f'' = 6x + \frac{1}{x^2} > 0$ for $x>0$. $\Rightarrow f''(1/\sqrt[3]{3}) > 0$. $\Rightarrow f(1/\sqrt[3]{3})$ local minimum.

(d)

x	$\searrow 0$	$1/\sqrt[3]{3}$	1
f	$+\infty$	$\frac{1}{3}(1+\ln 3) \doteq .7$	1

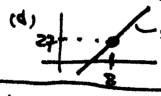
$\Rightarrow$ global minimum: $f(1/\sqrt[3]{3}) = \frac{1}{3}(1+\ln 3)$
no global maximum.

2 (20pts) (a) $8^{1/3} + 27^{1/3} = 2+3=5$, $27 - 3(8) + 2 = 5$. $\Rightarrow (8, 27)$ on $x^{1/3} + y^{1/3} = y - 3x + 2$.

(b) $\frac{1}{3}x^{-\frac{2}{3}} + \frac{1}{3}y^{-\frac{2}{3}}\frac{dy}{dx} = \frac{dy}{dx} - 3$. $\Rightarrow (\frac{1}{3}y^{-\frac{2}{3}} - 1)\frac{dy}{dx} = -(\frac{1}{3}x^{-\frac{2}{3}} + 3)$

$\Rightarrow \frac{dy}{dx} = -\frac{x^{-2/3}+9}{y^{-2/3}-3}$ (c) $\frac{dy}{dx}\Big|_{x=8} = -\frac{8^{-2/3}+9}{27^{-2/3}-3} = -\frac{\frac{1}{2^2}+9}{\frac{1}{3^2}-3}$ $\frac{333}{104} \doteq 3.2$

(d)



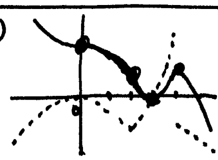
3 (25pts) (a) i. $\lim_{x\to\pi} \frac{x\sin x}{x-\pi} = \lim_{x\to\pi} \frac{\sin x + \pi\cos x}{1} = -\pi$ ii. $\lim_{x\to\infty} xe^{-2x} = \lim_{x\to\infty} \frac{x}{e^{2x}} = \lim_{x\to\infty} \frac{1}{2e^{2x}} = 0$

(b) i. $\int \frac{x^2+2x}{x}\,dx = \int (x+2)\,dx = \frac{1}{2}x^2 + 2x + C$

ii. $\int (\sin x + \cos x)^2 dx = \int (\sin^2 x + 2\sin x\cos x + \cos^2 x)\,dx = \int (1 + 2\sin x\cos x)\,dx$

$= \int 1\,dx + \int 2\sin x\cos x\,dx = x + \sin^2 x + C$

4 (15pts)



c.pt: 0, 3, 4
local max: $x=4$ local min: $x=3$
inflection point: $x=0, 2, 4$
concave up: $(-\infty, 0)$ $(2, 4)$
concave down: $(0, 2)$ $(4, +\infty)$

5 (20pts)

Volume $= \pi r^2 h = 10 \Rightarrow h = \frac{10}{\pi r^2}$

Cost: $f(r) = 2r\pi h(2) + \pi r^2(2) + \pi r^2(4) = \frac{40}{r} + 6\pi r^2$

$f'(r) = -\frac{40}{r^2} + 12\pi r = \frac{-40+12\pi r^3}{r^2} = 0 \Rightarrow 12\pi r^3 = 40$

$\Rightarrow r = \sqrt[3]{\frac{40}{12\pi}} = 1.03$ cm. Since $f \to \infty$ as $h \to 0$ and $r \to \infty$.

$f(1.03)$ must be minimum. $r = 1.03$, $h = 3$. cm