Name:______________________________ Score:_________

Instructions: You must show supporting work to receive full and partial credits.

1(20pts) (a) A function f is given at the those x values shown in the table.

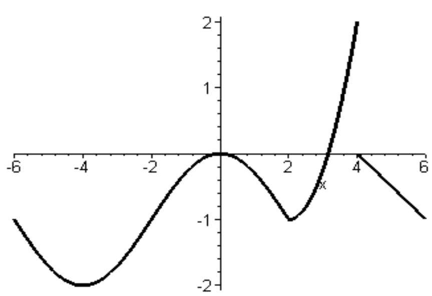
x	0.8	0.9	1	1.1	1.2
f(x)	1.28	1.62	2	2.42	2.88

i. Use difference quotients to approximate the 1st derivative $f'(1)$.

ii. Use the tangent line approximation to estimate $f(0.95)$.

(b) On the graph f

i. Label the points where the derivatives are zero as $A_1, A_2, ...$
ii. Label the points where the derivatives do not exist as $B_1, B_2, ...$
iii. Label the points where the 2nd derivatives are zero as $C_1, C_2, ...$
iv. List all the intervals on which f is increasing.
v. List all the intervals on which f is concave up.



(c) Use the definition of derivative to show that $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$.

(*Continue on Next Page ...*)

2(25pts) (a) How many partitioning subintervals are needed over interval $[0, 2]$ in order to have an approximating Riemann sum within 0.01 of the exact definite integral $\int_0^2 \frac{1}{\sqrt{8-x^2}}dx$?

(b) Approximate the integral $\int_0^2 \frac{1}{\sqrt{8-x^2}}dx$ by the right Riemann sum with the same partition number you found in (a).

(c) Use your answer in part (b) to estimate the average value of $f(x) = \frac{1}{\sqrt{8-x^2}}$ in the interval $[0, 2]$.

(d) Does the Trapezoidal Sum over estimate or under estimate the exact definite integral? Explain your answer with a graph.

(*Continue on Next Page ...*)

3(30pts) Find the derivative of each function.

(a) $3e^{5x+\sin x}$

(b) $\dfrac{2}{x^2} + \sqrt{x^2+1}$

(c) $\dfrac{\sin(x+100)}{x^3+x+1}$

(d) Find the second derivative $f''(x)$ of $f(x) = \cos(e^x)$.

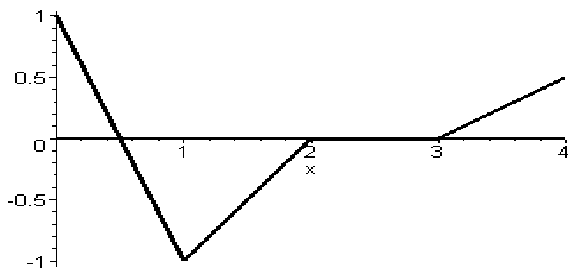
(e) Find the exact value of $f'(\pi)$, if $f(x) = x^2 \sin x$.

(*Continue on Next Page ...*)

4(25pts) (a) Use the left sum to estimate the definite integral $\int_1^2 f(x)dx$ for the function given in the table

x	1	1.2	1.4	1.6	1.8	2
f(x)	1.28	1.62	2	2.42	2.88	3.40

(b) The derivative F' of a function is given in the graph. Assume $F(0) = 1$, use the Fundamental Theorem of Calculus to find these values of F:



(i) $F(0.5)$

(ii) $F(1)$

(iii) $F(2.5)$

(iv) $F(4)$

(c) Find the exact value of $\int_0^e (2x - 1)dx$, using the Fundamental Theorem of Calculus.

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3 Bonus Points: True or false?

(a) You cannot integrate a function if it has a discontinuous jump even though it is continuous everywhere else.

(b) You can differentiate a function so long as it is continuous.

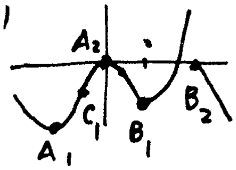
The End

Math 106 Exam II Solution Key Spring 2002

1 (a) (i) $f'(1) \doteq \frac{f(1.1)-f(1)}{1.1-1} = \frac{2.42-2}{.1} = 4.2$ (or $f'(1) \doteq \frac{f(.9)-f(1)}{.9-1} = 3.8$)

(ii) $f(0.95) \doteq f(1) + f'(1)(0.95-1) = 2 + 4.2(-.05) = 1.79$ (or. 1.81)

(b)



iv. $f \uparrow$ on $[-4, 0]$, $[2, 4]$

v. $f \cup$ on $[-6, -2]$, $[2, 4]$

(c) $\frac{d}{dx}\left(\frac{1}{x}\right) = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{(x+h)x}}{h} = \lim_{h\to 0} \frac{-h}{h(x+h)x} = \lim_{h\to 0} \frac{-1}{(x+h)x} = \frac{-1}{x^2}$

2. (a) $f(x) = \frac{1}{\sqrt{8-x^2}}$ $\nearrow \Rightarrow \left| \int_0^2 f(x)dx - \text{Riemann sum of regular n-partition} \right| \le |f(2)-f(0)| \frac{2-0}{n} \le 0.01$

$\Rightarrow n \ge \frac{|f(2)-f(0)| 2}{.01} = 29.29$, $\Rightarrow n = 30$

(b) $\int_0^2 \frac{1}{\sqrt{8-x^2}} dx \doteq \sum_{i=1}^{30} f(x_i)\Delta x = 0.79$

(c) $\bar{f} = \frac{1}{2-0}\int_0^2 \frac{1}{\sqrt{8-x^2}} dx \doteq \frac{0.79}{2} = 0.395$

(d)

Over estimate because $f(x)$ is concave-up in $[0, 2]$

3. (a) $(3e^{5x+\sin x})' = 3e^{5x+\sin x} \cdot (5+\cos x)$

(b) $\left(\frac{2}{x^2} + \sqrt{x^2+1}\right)' = (2x^{-2} + (x^2+1)^{\frac{1}{2}})' = -4x^{-3} + \frac{1}{2}(x^2+1)^{\frac{1}{2}-1} \cdot 2x = -\frac{4}{x^3} + \frac{x}{\sqrt{x^2+1}}$

(c) $\left(\frac{\sin(x+100)}{x^3+x+}\right)' = \frac{\cos(x+100)\cdot(x^3+x+1) - \sin(x+100)\cdot(3x+1)}{(x^3+x+1)^2}$

(d) $f'(x) = (\cos(e^x))' = -\sin(e^x).e^x$. $f'' = -[\cos(e^x)\cdot e^{2x} + \sin(e^x)\cdot e^x]$

(e) $f'(x) = (x^2\sin x)' = 2x\sin x + x^2\cos x$, $f'(\pi) = 2\pi\sin\pi + \pi^2\cos\pi = -\pi^2$

4. (a) $\int_1^2 f(x)dx \doteq (1.28 + 1.62 + 2 + 2.42 + 2.88)\cdot(0.2) = 2.04$

(b)(i) $F(0.5) = F(0) + \int_0^{0.5} F'(x)dx = 1 + \frac{1}{2}(.5)(1) = \frac{5}{4}$ (ii) $F(1) = 1$

(iii) $F(2.5) = 1 + \frac{1}{2}(-1)(1) = 0.5$ (iv) $F(4) = .5 + \frac{1}{2}\frac{1}{2}(1) = \frac{3}{4}$

(c) $\int_0^e (2x-1)dx = x^2 - x \Big|_0^e = e^2 - e = e(e-1)$

Bonus. (a) False (b) False