Math 106 Sec.551–555	Hour Exam 3	Nov. 30, 2000

Name:____________________________ **Score:**____________

Instructions: You must show supporting work to receive full and partial credits.

1(20pts) For function $f(x) = x^3 - 3x^2 - 9x + 26, 0 \leq x \leq 4$, find the values of x for which

(a) $f(x)$ has a local maximum or a local minimum;

(b) $f(x)$ has a global maximum or global minimum.

~~Other~~ Alternative problems:

find local extrema of

i) $f(x) = 3x^2(x-2)$

ii) $g(x) = \dfrac{x - x^2}{2+x}$

Find global max, min of

i) $f(x) = \dfrac{1}{\sqrt{3}\sin x + \cos x}$

$0 \leq x \leq 2\pi$

ii) $g(x) = x^3 - 4x^2 + 4x, 0 \leq x \leq 4$

2(20pts)

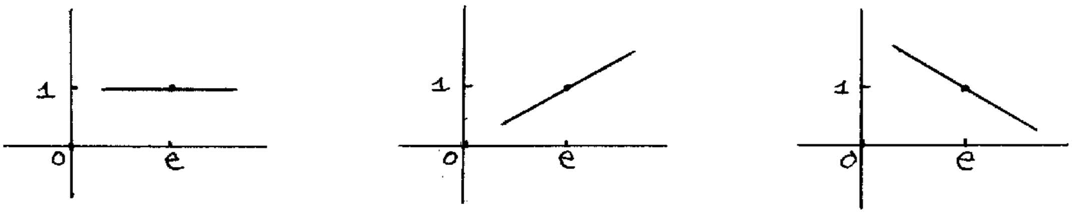
(a) Verify that the point $(e, 1)$ is on the curve $y = f(x)$ defined by the equation

$$x \ln y + y^3 = e^{y-1}.$$

Practice problems.
Find $\frac{dy}{dx}$ of
i) $\cos^2 y + x \ln y = y + 2$
ii) $\sqrt{x} + \sqrt{y} = xy - 1$
at point $(4, 1)$.
Then find the tangent line approximation of the function $y = y(x)$ at $(4, 1)$ for problem ii).

(b) Find the derivative $\frac{dy}{dx}$ of the function $y = f(x)$ defined by the equation in (a) above.

(c) Circle the graph that best approximates the curve $y = f(x)$ at the point $(e, 1)$, where $y = f(x)$ is determined by the equation in (a) above.



3(20pts)

(a) Use L'Hopital's Rule to find the limit $\lim_{x \to 1} \frac{\ln x}{x-1}$

Additional ~~Extra~~ problems

Find

i) $\lim_{x \to \infty} x \sin \frac{1}{x}$

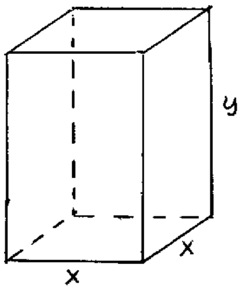
ii) $\lim_{x \to 0} \frac{\sqrt{1 - \cos x}}{x}$

iii) $\lim_{x \to \infty} \frac{\ln(x^5 + x)}{x}$

iv) $\lim_{x \to 0} x \ln x$

(b) Verify the identity $\cosh(2x) = \cosh^2 x + \sinh^2 x$.

4(20pts) A square-bottomed box with no top has a fixed volume 10 m^3. What dimensions minimize the surface area?



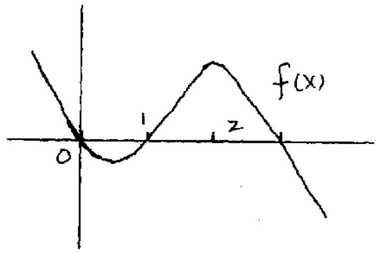
~~Other~~ Additional Problems.

i) Find the dimensions of a rectangular beam that is cut from a cylindrical log of radius 30cm and has the maximum cross-section area.

ii) Find the coordinates of the point on the parabola $y=x^2$ which is closest to the point $(3,0)$. (Hint: minimize the distance squared).

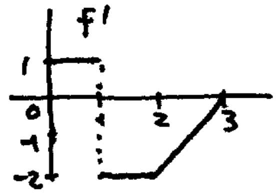
iii) A landscape architect plans to enclose a 5000 square foot rectangular region. She will use shrubs costing \$25 per foot along two opposite ~~adjacent~~ sides and fencing costing \$20 per foot along the other two sides. Find the minimum cost.

5(20pts) (a) A function $f(x)$ is given graphically as follows. Sketch an antiderivative $F(x) = \int f(x)dx$ with $F(1) = 0$. Identify local max, local min, inflection pts.

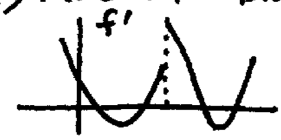


· Additional Problems.

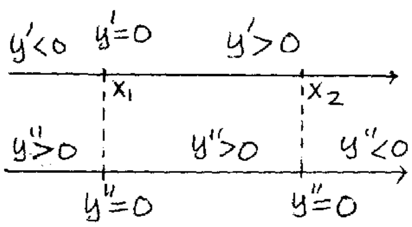
i) Find $f(0)$, $f(2)$, $f(3)$ if $f(1) = 0$ and f' as follows



ii) Find inflection pts of f given f'



(b) Sketch a possible graph of $y = f(x)$, using the given information about the derivative y' and the second derivative y''.



6 (10pts). Find $\int \left(t\sqrt{t} + \frac{1}{t\sqrt{t}} + \frac{5}{e^{3t}} \right) dt$

Additional problems.

Find:

i) $\int 7 \ln(t+5)\, dt$

ii) $\int \frac{7}{3x+1}\, dx$

iii) $\int \frac{(y^2+1)^2}{y}\, dy$

iv) $\int \frac{(\sin t + \cos t)^2}{\sin t}\, dt$

THE END

Solution Key to Sample Exam 3

1. (a) $f(x)=x^3-3x^2-9x+26$, $f'=3x^2-6x-9=$ [redacted], $x_{1,2}=\frac{6\pm\sqrt{36+4(3)(9)}}{2(3)}$
$=-1, 3.$ $x_2=3$ in $[0,4]$. 1st derivative test

x	$(0\ 3)$	$(3\ 4)$
f'	<0	>0

$\Rightarrow (3, f(3))$ local minimum point

(or alternatively, by 2nd derivative test, $f''(3)=6x-6\,|_{x=3}>0 \Rightarrow$ loc. min)

(b)

x	0	3	4
f	26	-1	6.

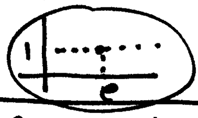
$\Rightarrow$ global max. $f(0)=26$
global min. $f(3)=-1$

2 (a) $x\ln y+y^3=e^{y-1}$, $e\ln 1+1^3=1=e^{1-1}$. $\Rightarrow (e,1)$ is on the curve

(b) $\frac{d}{dx}(x\ln y+y^3)=\frac{d}{dx}(e^{y-1}) \Rightarrow 1\cdot\ln y+x\frac{1}{y}\frac{dy}{dx}+3y^2\frac{dy}{dx}=e^{y-1}\cdot\frac{dy}{dx}$

$\Rightarrow \left(\frac{x}{y}+3y^2-e^{y-1}\right)\frac{dy}{dx}=-\ln y$, $\Rightarrow \frac{dy}{dx}=-\frac{\ln y}{\frac{x}{y}+3y^2-e^{y-1}}=0$ at $(x,y)=(e,1)$.

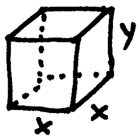
(c)



3. (a) $\frac{0}{0}$ type. $\lim_{x\to1}\frac{\ln x}{x-1}=\lim_{x\to1}\frac{\frac{1}{x}}{1-0}=1.$

(b) $\cosh(2x)=\frac{e^{2x}+e^{-2x}}{2}$. $\cosh^2x+\sinh^2x=\left(\frac{e^x+e^{-x}}{2}\right)^2+\left(\frac{e^x-e^{-x}}{2}\right)^2$
$=\frac{e^{2x}+2e^x\cdot e^{-x}+e^{-2x}}{4}+\frac{e^{2x}-2e^xe^{-x}+e^{-2x}}{4}=\frac{2e^{2x}+2e^{-2x}}{4}=\frac{e^{2x}+e^{-2x}}{2}=\cosh(2x)$

4.

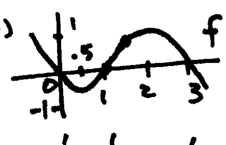


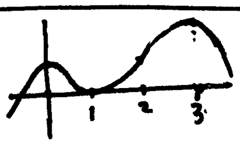
Volume $=x^2y=10$. Surface area $A=x^2+4xy$ (w/out top)
$\Rightarrow y=\frac{10}{x^2}$ and $A(x)=x^2+4x\cdot\frac{10}{x^2}=x^2+\frac{40}{x}$. $A'(x)=2x-\frac{40}{x^2}=0$
$\Rightarrow 2x=\frac{40}{x^2}$, $2x^3=40$, $x^3=20$, $x=\sqrt[3]{20}$, $y=\frac{10}{\sqrt[3]{400}}$

x	0	$\sqrt[3]{20}$	∞
A	∞	min.	∞

$\Rightarrow (x,y)=\left(\sqrt[3]{20}, \frac{10}{\sqrt[3]{400}}\right)\approx(2.714, 1.357)$ meters.

5. (a)





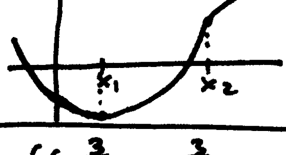
local max: 0, 3
local min: 1
inflection pt. .5, 2

(b)

$y'<0$ $y'=0$ $y'>0$, x_1

$y''>0$ $y''=0$ $y''<0$, x_1, x_1

$y''=0$ $y''=0$



6. $\int\left(t\sqrt{t}+\frac{1}{t\sqrt{t}}+\frac{5}{e^{3t}}\right)dt=\int\left(t^{\frac{3}{2}}+t^{-\frac{3}{2}}+5e^{-3t}\right)dt$
$=\int t^{\frac{3}{2}}dt+\int t^{-\frac{3}{2}}dt+5\int e^{-3t}dt=\frac{1}{\frac{3}{2}+1}t^{\frac{3}{2}+1}+\frac{1}{-\frac{3}{2}+1}t^{-\frac{3}{2}+1}+5\frac{1}{-3}e^{-3t}+C$
$=\frac{2}{5}t^{\frac{5}{2}}-2\frac{1}{\sqrt{t}}-\frac{5}{3}e^{-3t}+C$