Math 106 Sec.551–555 **Hour Exam 2** **Nov. 2, 2000**

Name:____________________________ **Score:**____________

Instructions: You must show supporting work to receive full and partial credits.

1(20pts)

(a) How many partitioning subintervals are needed over interval $[1, 2]$ in order to have an approximating Riemann sum within 0.01 of the exact definite integral $\cancel{\int_1^2 e^{x^2}\,dx}$? $\int_1^2 e^{-x^2}dx$

(b) Approximate the integral $\cancel{\int_1^2 e^{x^2}\,dx}$ $\int_1^2 e^{-x^2}dx$ by the left Riemann sum with the same partition number you found in (a).

(c) Is the left sum approximation an under-estimate or an over-estimate of the true value? Explain your answer.

2(20pts)

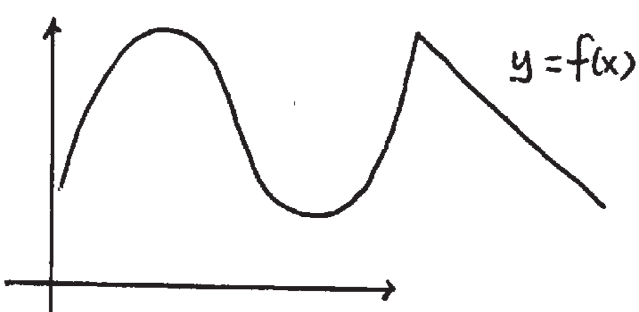
(a) Use the definition of derivative to show that $(x^2)' = 2x$.

(b) The following table defines a function f at a few point. Approximate $f'(1.1), f'(1.20), f''(1.2)$ and fill in the blank boxes with your answers.

x	1.1	1.2	1.3
f(x)	1.21	1.44	1.69
f'(x)			
f''(x)			

(c) Label points A, B, C, D on the graph f with the following description

(i) Point A is a point on the curve where the derivative is zero.
(ii) Point B is a point on the curve where the function f is not differentiable.
(iii) Point C is a point on the curve where the derivative is constant in an interval containing C.
(iv) Point D is a point on the curve where the second derivative f'' is zero.



3(20pts) **Find the derivative of**

(a) $\ln(1 - \cos x)$

(b) $3^x + 3x^3$

(c) $e^{-2x} \sin x$

(d) $\dfrac{1 - x^2}{1 + x^2}$

4(20pts) In (a) and (b), find the exact value of the definite integrals by the Fundamental Theorem of Calculus

(a) $\int_0^1 4x^3 dx$.

(b) $\int_0^2 e^{2x} dx$.

(c) Given that $\int_1^2 f(x)dx = 2$, $\int_2^4 f(x)dx = -1$, $\int_1^2 g(x)dx = 0$, $\int_2^4 g(x)dx = 5$. Fill in the following blanks.

(i) $\int_1^4 f(x)dx =$ ________.

(ii) $\int_1^2 [3f(x) - 2g(x)]dx =$ ________.

(iii) $\int_1^3 f(x)dx + \int_3^4 f(x)dx =$ ________.

(iv) $\int_4^2 f(x)dx =$ ________.

5(20pts)

(a)

Given $\begin{cases} f(1) = -1, \; g(1) = 4 \\ f'(1) = 3, \; g'(1) = 2 \\ f(2) = 3, \; g(2) = 1 \\ f'(2) = 0, \; g'(2) = -2 \end{cases}$ Find $\begin{cases} \text{(i)} \;\; h'(2), & \text{if } h(x) = 2f(x)g(x). \\ \text{(ii)} \;\; h'(2), & \text{if } h(x) = f(g(x)). \\ \text{(iii)} \;\; h'(1), & \text{if } h(x) = \dfrac{f(x)}{g(x)}. \\ \text{(iv)} \;\; h'(1), & \text{if } h(x) = f(g(x)/2). \end{cases}$

(i)

(ii)

(iii)

(iv)

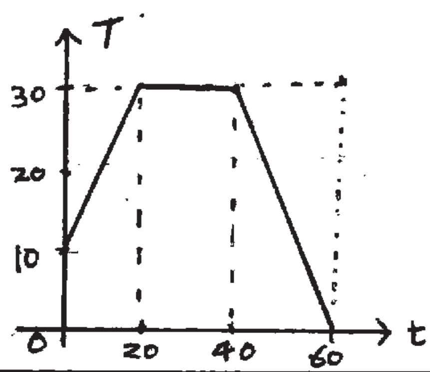
(b) An economist is interested in how the price of a certain item affects its sales. Suppose that at a price of $\$p$, a quantity, q, of the item is sold. If $q = f(p)$, explain the meaning of each of the following statements

(i) $f(150) = 2000$ (ii) $f'(150) = -25$

(i)

(ii)

(c) The temperature $T(t)$ in °C as a function of time t in minute is given by the graph. Estimate the average temperature over the time interval $[0, 60]$ and label it on the T-axis.



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Sample Test 2 Soln Key

1 (a) $f(x)=e^{-x^2}$ monotone decreasing. Use regular partition of n subinterval. then $\Delta x = \frac{b-a}{n} = \frac{2-1}{n} = \frac{1}{n}$. And

$$\left|\int_1^2 e^{-x^2}dx - \text{Riemann sum of regular partition } n\right| \le |f(b)-f(a)|\Delta x = \frac{|e^{-4}-e^{-1}|}{n} \le 0.01$$

$$\Rightarrow n \ge \frac{e^{-1}-e^{-4}}{0.01} = 100(e^{-1}-e^{-4}) = 34.956. \Rightarrow \boxed{n=35}$$

(b) $\int_1^2 e^{-x^2}dx \approx 0.14$

(c) Over-estimate because f is decreasing

2 (a) $$(x^2)' = \lim_{h\to0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to0}\frac{(x+h)^2-x^2}{h} = \lim_{h\to0}\frac{(x+h-x)(x+h+x)}{h}$$

$$=\lim_{h\to0}\frac{h\cdot(2x+h)}{h} = \lim_{h\to0}(2x+h) = 2x$$

(b)

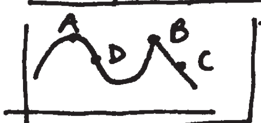
x	1.1	1.2	1.3
f	1.21	1.44	1.69
f'	2.3	2.5	
f''		2	

$$f(1.1) \doteq \frac{f(1.2)-f(1.1)}{1.2-1.1} = \frac{1.44-1.21}{0.1} = \frac{.23}{.1} = 2.3$$

$$f(1.2) \doteq \frac{f(1.3)-f(1.2)}{1.3-1.2} = \frac{1.69-1.44}{.1} = \frac{.25}{.1} = 2.5$$

$$f''(1.2) \doteq \frac{f'(1.1)-f'(1.2)}{1.1-1.2} = \frac{2.3-2.5}{-.1} = \frac{-.2}{-.1} = 2.$$

(c)



3 (a) $$(\ln(1-\cos x))' = \frac{1}{1-\cos x}\cdot(1-\cos x)' = \frac{1}{1-\cos x}(0-(-\sin x)) = \boxed{\frac{\sin x}{1-\cos x}}$$

(b) $(3^x+3x^3)' = \boxed{3^x\ln 3+9x^2}$ (c) $(e^{-2x}\sin x)' = (e^{-2x})'\sin x + e^{-2x}(\sin x)' = \boxed{-2e^{-2x}\sin x + e^{-2x}\cos x}$

(d) $$\left(\frac{1-x^2}{1+x^2}\right)' = \frac{(1-x^2)'(1+x^2)-(1-x^2)(1+x^2)'}{(1+x^2)^2} = \frac{-2x(1+x^2)-2x(1-x^2)}{(1+x^2)^2} = \boxed{\frac{-4x}{(1+x^2)^2}}$$

4 (a) $\int_0^1 4x^3dx = x^4\Big|_0^1 = 1^4-0^4 = 1$ (b) $\int_0^2 e^{2x}dx = \frac12 e^{2x}\Big|_0^2 = \frac12(e^4-1)$

(c) (i) $\int_1^4 f(x)dx = \int_1^2 f dx + \int_2^4 f dx = 2+(-1) = 1$ (ii) $\int_1^2(3f-2g)dx$
$= 3\int_1^2 f dx - 2\int_1^2 g\,dx = 3(2)-2(0) = 6$ (iii) $\int_1^3 f dx + \int_3^4 f(x)dx = \int_1^4 f dx = 1$

(iv) $\int_4^2 f(x)dx = -\int_2^4 f(x)dx = -(-1) = 1$

5 (a) (i) $h'(2) = 2(f'(2)g(2)+f(2)g'(2)) = 2(0(1)+3(-2)) = -12$

(ii) $h'(2) = f'(g(2))g'(2) = f'(1)g'(2) = 3(-2) = -6$

(iii) $h'(1) = \frac{f'(1)g(1)-f(1)g'(1)}{g^2(1)} = \frac{3(4)-(-1)(2)}{4^2} = \frac{14}{16} = \frac{7}{8}$

(iv) $h'(1) = f'(g(1)/2)\,g'(1)/2 = f'(1)g'(1)/2 = 3(2)/2 = 3$

(b) (i) 2000 items sold at unit price \$150 each (ii) For \$1 increase in sale price, 25 items less are sold.

(c) $$\bar{T} = \frac{1}{60}\int_0^{60}T(t)dt = \frac{1}{60}\left[\frac{10+30}{2}\cdot20+30(20)+\frac{30}{2}(20)\right] \doteq 21.7$$