Name: _____   **Score:** _____

Instructions: You must show supporting work to receive full and partial credits.

1(20pts)   (a) Find the domain of $f(x) = \dfrac{x-3}{x^2-x}$ and solve $f(x) = 0$.

(b) Find the vertical asymptote of $g(x) = \dfrac{1-4x}{x+2}$.

(c) Determine the behaviour of $h(x) = \dfrac{x + \ln x + 10}{x^{3/2} - 10}$ as $x \to +\infty$ and find the horizontal asymptote.

2(20pts) (a) Simplify the expression as much as possible

$$\ln(10e) - \ln 5 + \log(e^{x \ln 10}).$$

(b) Solve for x from $10^{x+3} = 5e^x$.

(c) Circle the dominating function as $x \to +\infty$ in each pair.

(i) $1.1^{0.1x}, \quad x^{1.1} + 1,000$ (ii) $\sqrt{x} + 1,000 \ln x, \quad \log x$ (iii) $\log x, \quad \ln x$
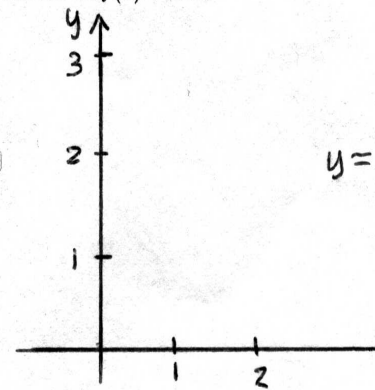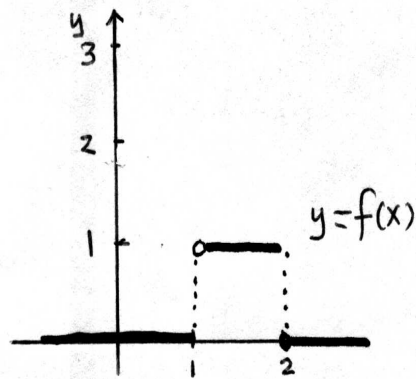
3(20pts) (a) Let $1 \leq x < \infty$ be the domain of $y = f(x) = \frac{1}{\ln x + 1}$. What is the range?

(b) Explain why $y = f(x) = \frac{1}{\ln x + 1}$ with domain $1 \leq x < \infty$ is invertible.

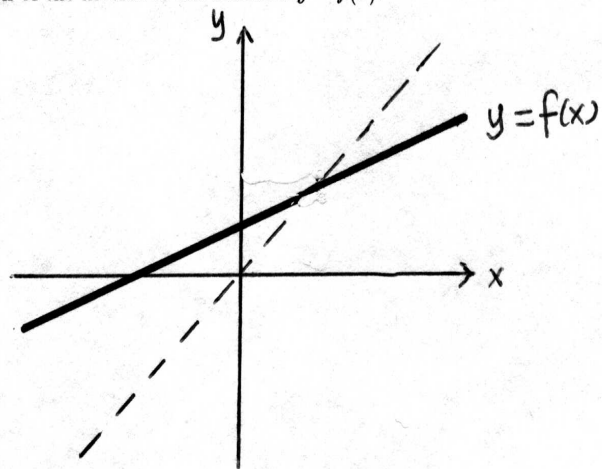(c) Find the inverse $y = f^{-1}(x)$.

4(20pts)

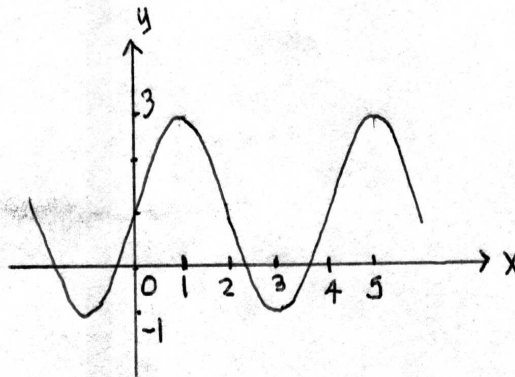(a) The graph of function $y = f(x)$ is given. Sketch $1.5f(x) + 0.5$.

$y = f(x)$

$y = 1.5f(x) + 0.5$

(b) Sketch the graph of the inverse of the function $y = f(x)$.

$y = f(x)$

(c) Find a formula for the graph given.

5(20pts)   (a) Find the equation for the line through $(0, 1)$ and $(2, -1)$.

(b) If a line is parallel to $2x + 3y - 1 = 0$, what is the slope of the line?

(c) Circle all the even functions

$$x^{10} + x^2, \qquad x^6 + 1, \qquad x^2 \cos(2x), \qquad x^3 + \cos x, \qquad 2^{2x} + x^2$$

Sample Exam Solu. Key, Math 106

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1. (a) $f(x) = \frac{x-3}{x^2-x}$, Domain = {all x except $x=0$, $x=1$}

   $f(x) = 0 \Rightarrow x-3 = 0 \Rightarrow \boxed{x = 3}$

   (b) $g(x) = \frac{1-4x}{x+2}$, vertical asymptote, $\boxed{x = -2}$

   (c) $h(x) = \frac{x + \ln x + 10}{x^{3/2} - 10} \approx \frac{x}{x^{3/2}} = \frac{1}{x^{1/2}} \to 0$ as $x \to +\infty$.

   horizontal asymptote: $\boxed{y = 0}$

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2. (a) $\ln(10e) - \ln 5 + \log(e^{x \ln 10}) = \ln \frac{10e}{5} + \log(e^{\ln 10^x})$

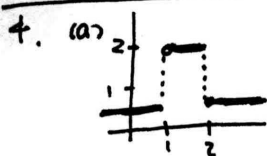 $= \ln 2 + 1 + \log 10^x = \boxed{\ln 2 + 1 + x}$

   (b) $10^{x+3} = 5e^x$, $\ln 10^{x+3} = \ln 5 + \ln e^x$ $(x+3)\ln 10 = \ln 5 + x$

   $\Rightarrow (\ln 10 - 1)x = \ln 5 - \ln 10^3$, $\Rightarrow x = \frac{-\ln \frac{1000}{5}}{\ln 10 - 1} = \boxed{\frac{\ln 200}{1 - \ln 10}}$

   (c) (i) $\boxed{1.1^{0.1x}}$ (ii) $\boxed{\sqrt{x} + 1{,}000 \ln x}$ (iii) $\boxed{\ln x}$
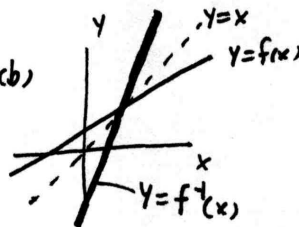
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3. (a) $y = f(x) = \frac{1}{\ln x + 1}$. Domain = $[1, +\infty)$. Range = $(0, 1]$.

   (b) $\ln x \uparrow$, $\ln x + 1 \uparrow$, $\frac{1}{\ln x + 1} \downarrow$. $\Rightarrow f(x)$ invertible

   (c) $y = \frac{1}{\ln x + 1}$, $\frac{1}{y} = \ln x + 1$ $\ln x = \frac{1}{y} - 1 = \frac{1-y}{y}$, $x = e^{\frac{1-y}{y}}$.

   $\Rightarrow f^{-1}(x) = \boxed{e^{\frac{1-x}{x}}}$

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4. (a)

   $1.5 f(x) + .5$ (b)

   (c) $2 \sin\left(\frac{\pi}{2} x\right) + 1$.

   Amplitude = 2
   Period = $4 \left(= \frac{2\pi}{\pi/2}\right)$

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5. (a) Points $(0,1)$, $(2,-1)$. Slope $= \frac{-1-1}{2-0} = -1$. line $y - 1 = -1(x-0)$

   $\Rightarrow \boxed{y = 1-x}$

   (b) Any line parallel to $2x + 3y - 1 = 0$ has slope $= \boxed{-\frac{2}{3}}$

   (c) Even functions: $x^{10} + x^2$, $x^6 + 1$, $x^2 \cos(x)$,