Print Your Name: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯   **TA's Name:** ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

Instructions: You must show supporting work as much as possible to receive full and partial credits. No text book, notes, formula sheets allowed.

1. (**10 pts**) Water is being pumped into a vertical cylinder of radius 5 meters and height 20 meters at a rate of 3 cubic meters per minute. How fast is the water level rising? Use exact value and unit for your answer.

2. (**15 pts**) You must show all supporting work to receive credit for this problem. Consider the function $f(x) = (x^3 - 8)^2$.

   (a) Find all critical points of f.

   (b) Use the First Derivative Test to classify the critical points as local minima, local maxima, or neither.

3. (**15 pts**) The figure shows $f(x)$ and its tangent line at $x = a$.

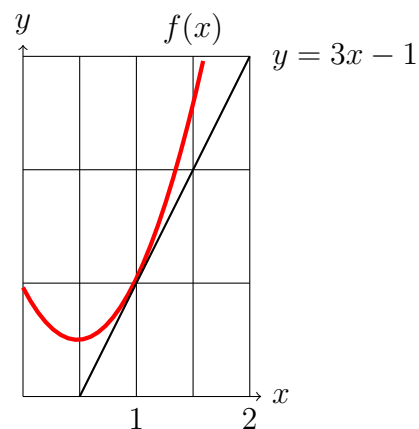   (a) What is a?

   (b) What is $f(a)$?

   (c) What is $f'(a)$?

   (d) What is the linearization?

   (e) Is the linear approximation an under- or overestimate?

   (f) Use the linearization to approximate the value of $f(1.2)$.

4. (**15 pts**) Consider the curve that is defined by the equation
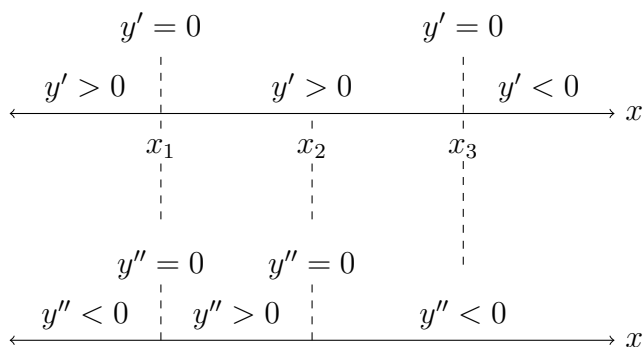
$$e^{2x} + 2xy + y^3 = 2$$

   (a) Use implicit differentiation to find $\dfrac{dy}{dx}$.

   (b) Verify that point $(0, 1)$ is on the curve.

   (c) Find an equation of the tangent line to the curve at the point $(0, 1)$.

5. (**15 pts**) Sketch a possible graph of $y = f(x)$ using the given information about the derivatives $y' = f'(x)$, $y'' = f''(x)$, and the information that $f(x_1) = 0$.

$$y' = 0 \qquad\qquad y' = 0$$

$$y' > 0 \qquad\qquad y' > 0 \qquad\qquad y' < 0$$

$$\xrightarrow{\hspace{10cm}} x$$

$$x_1 \qquad\qquad x_2 \qquad\qquad x_3$$

$$y'' = 0 \qquad y'' = 0$$

$$y'' < 0 \qquad y'' > 0 \qquad\qquad y'' < 0$$

$$\xrightarrow{\hspace{10cm}} x$$

6. (**15 pts**) Find the absolute maximum and the absolute minimum points for the function $f(x) = \dfrac{\ln x}{x}$ on the interval $[1, 4]$. (You must show work to receive points. Calculator read-out answer will not be accepted.)

7. (**15 pts**) A boat sails on a straight line path. A lighthouse is located 1 km West and 2 km South of the course. Find the location of the boat that is nearest the lighthouse. (You must show work to receive credit. Also your answer must be in exact values. For example, 1.4142... will not be accepted for the exact value $\sqrt{2}$.)

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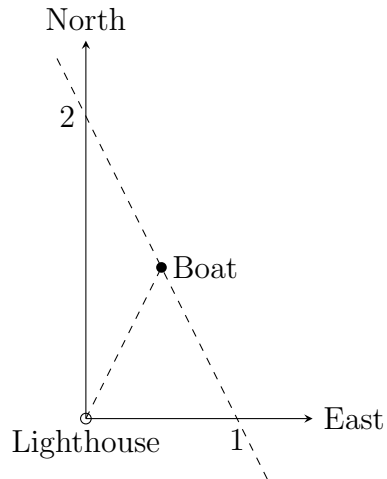
2 point Bonus Question: The state flower of Nebraska is (a) goldenrod   (b) dandelion   (c) sunflower (d) rose