Solution Key to Test 1. Math 106, Fall '08

1 (18 pts) (a) $\lim_{x\to 1} \frac{2x-2}{x^2+x-2} \overset{\frac{0}{0}}{=} \lim_{x\to 1} \frac{2(x-1)}{(x-1)(x+2)} = \lim_{x\to 1} \frac{2}{x+2} = \frac{2}{3}$

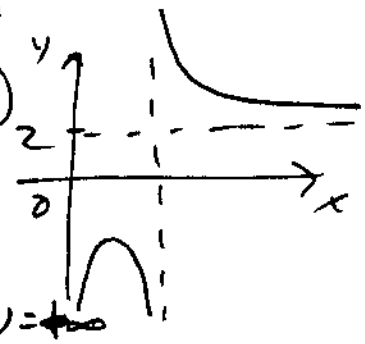
(b) $\lim_{x\to\infty} \frac{2x^{3/2}+x}{x\sqrt{x}+2} = \lim_{x\to\infty} \frac{(2x^{3/2}+x)/x^{3/2}}{(x^{3/2}+2)/x^{3/2}} = \lim_{x\to\infty} \frac{2+\frac{1}{x^{1/2}}}{1+2/x^{3/2}} = \frac{2+0}{1+0} = 2$

(c) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (3-x) = 3-2 = 1$.

2 (14 pts). (a) $f'(0.4) \approx \frac{f(0.6)-f(0.4)}{0.6-0.4} = \frac{1-0.8}{0.2} = 1$ or $\approx \frac{f(0.2)-f(0.4)}{0.2-0.4} = -0.5$

(b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h+1} - \frac{1}{x+1}}{h} = \lim_{h\to 0} \frac{\frac{x+1-(x+h+1)}{(x+h+1)(x+1)}}{h}$

$= \lim_{h\to 0} \frac{\frac{-h}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{-1}{(x+h+1)(x+1)} = \frac{-1}{(x+1)^2}$



3 (18 pts) $f(x) = \frac{2x^2+5x+2}{x^2-x}$, x int: $f(x)=0$, $2x^2+5x+2=0$, >0

which always >0 for $x>0$. $\lim_{x\to\infty} f(x) = 2$, $y=2$.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{2x^2+5x+2}{x(x-1)} = -\infty$, $\lim_{x\to 1^-} \frac{2x^2+5x+2}{x(x-1)} = -\infty$, $\lim_{x\to 1^+} f(x) = +\infty$

4 (16 pts) (a) $y' = 3x^2-12 = 3(x^2-4) = 0$, $x = \pm 2$ (b)

$$\begin{array}{r} 2x+1 \\ x-3\,\overline{)\,2x^2-5x-2} \\ \underline{2x^2-6x} \\ x-2 \\ \underline{x-3} \\ 1 \end{array}$$

$\Rightarrow \frac{2x^2-5x-2}{x-3} = 2x+1+\frac{1}{x-3}$. $\Rightarrow y = 2x+1$

5 (8 pts) (a) $g'(x) = 2(f(x)+1)^{2-1}\cdot(f'(x)+0) \overset{@x=2}{=} 2(9+1)\cdot 4 = 80$

(b) $y(x)$ is not cont. at $x=-1$, $\lim_{x\to -1} \frac{x^2-8x-9}{x+1} = \lim_{x\to -1} \frac{(x+1)(x-9)}{x+1} = \lim_{x\to -1} x-9 = -10$

exists. $\Rightarrow$ $x=-1$ is removable.

6 (8 pts) (a) $x' = t^{-1/2} - 1 \overset{t=4}{=} \frac{1}{\sqrt{4}} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$. speed $= \frac{1}{2}$ m/sec

(b) $x'' = -\frac{1}{2}t^{-3/2} \overset{t=4}{=} -\frac{1}{2}\frac{1}{(\sqrt{4})^3} = -\frac{1}{16}$ m/sec^2

7 (18 pts). (a) $f'(x) = (2x+\frac{1}{2}x^{-1/2})\cos x - (x^2+\sqrt{x})\sin x - 4\cos(x^2)\cdot 2x$

(b) $g'(x) = \frac{(2e^x+1)(e^x+1) - (2e^x+x)e^x}{(e^x+1)^2}$

(c) $h'(x) = 5e^{(2x+\sin x)^2}\cdot 2(2x+\sin x)\cdot(2+\cos x)$.

Bonus Points. (c) -