Math 106-Sec 45X Fall '04 **Exam 1** **Score:** _____

Name: _____ **TA's Name:** _____

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score							

Instructions: You must show supporting work to receive full and partial credits. No text book, notes, formula sheets allowed.

1(14pts) (7pts each) (a) Find the limit of

$$\lim_{x \to 1} \frac{x^2 + 2x - 3}{x - 1}$$

analytically. (Use long division to simplify. Numerical and graphical means are not accepted.)

(b) Estimate the length of the curve $y = x^3 + 2$ in the interval $0 \leq x \leq 1$ use $n = 4$ line segments.

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2(14pts) (7pts each) (a) Estimate the limit of the sequence $\lim_{n\to\infty} a_n = n^2 \cos\frac{1}{n} - n^2$, using $n = 1, 10, 100, \ldots, 10^6$.

(b) Estimate the slope of $f(x) = \sqrt{x+1}$ at $x = 2$.

3(12pts) (6pts each) (a) Find the exact solutions of $\sin^2 x - \sin x = 0$.

(b) Find the exact solution of $2\ln(4x) - 1 = 6$. (Comment: if π is part of the answer, an approximation such as 3.14... will not be accepted as an answer.)

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4(12pts) (6pts each) (a) Use the Intermediate Value Theorem to show that there is a zero point in the interval $[0, 1]$ for the function $f(x) = \cos x - x$. Then use the method of bisections to find an interval of length $1/8$ which contains the zero. (Make sure to verify all the conditions of the theorem.)

(b) Use the Squeeze Theorem to determine the limit $\lim_{x\to 0} x^3 \sin^2 \frac{1}{x}$. (Make sure to verify all the conditions of the theorem.)

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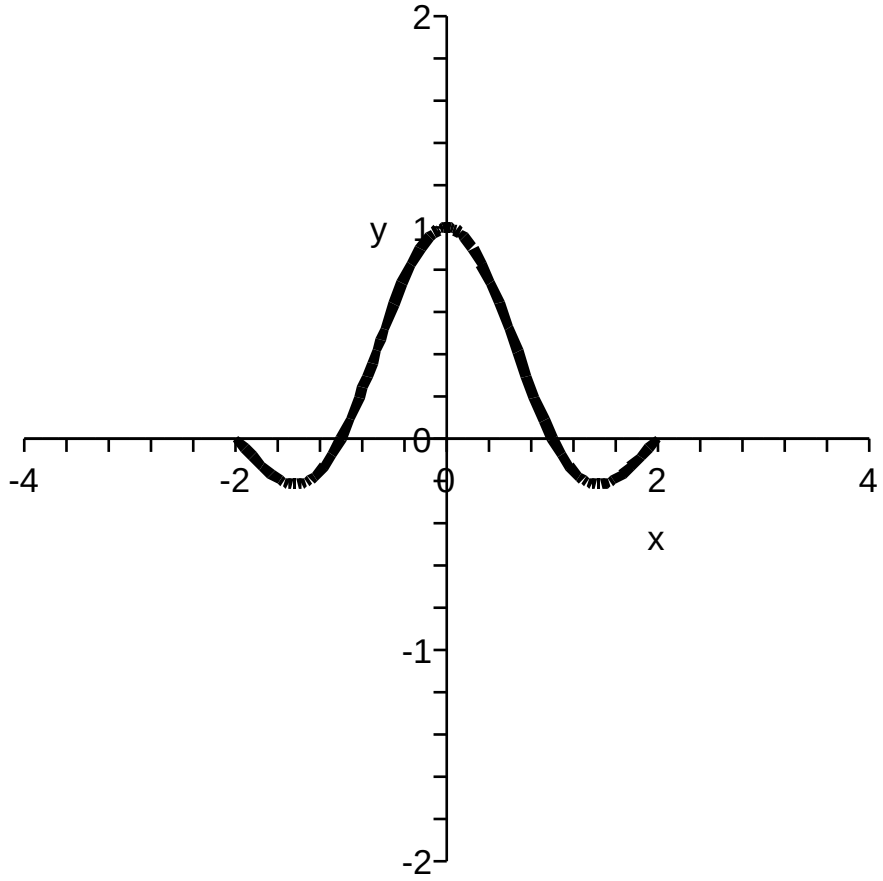
5(12pts) (4pts each) The graph of a function $y = f(x)$ is given below. Sketch the graphs of (a) $f(x+1)$, (b) $-2f(x)$, (c) $f(2x)$. (Label all graphs.)



6(12pts) Find the discontinuity of the function $f(x) = \dfrac{\sin x - 1}{x(x - \pi/2)}$. For each discontinuity that is removable, define a new function that removes the discontinuity. (Show your work, numerical or graphical, on all relevant limits.)

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7(12pts) (6pts each) Given the function $f(x) = \dfrac{x^2 - x - 2}{2x^2 - x - 6}$.

(a) Find all the horizontal asymptotes. (Show your work on all relevant limits.)

(b) Find all the vertical asymptotes. (Show your work on all relevant limits.)

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8(12pts) (6pts each) For the limit $\lim_{x\to 1}\sqrt{x+3}=2$, numerically and graphically determine a δ corresponding to the given value ϵ as in a proof of the limit according to the ϵ-δ definition:

(a) $\epsilon = 0.1$

(b) $\epsilon = 0.01$

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2 Bonus Points: Sir Isaac Newton made major discoveries in these areas (circle all that apply): (a) optics, (b) calculus, (c) physics, (d) evolution. (*... The End*)