# FINAL EXAM

Math 106, Fall Semester 2011

Name (Print):______________________________

Student ID Number:______________________________

TA Name:______________________________

Please Circle your professor's name:

Avalos  Geisbauer  Johnson  Rammaha  Rogge  Toundykov  True

Please Circle your class time:

7:30 a.m.  8:30 a.m.  9:30 a.m.  10:30 a.m.  11:30 a.m.  12:30 p.m  6:30 p.m.

**INSTRUCTIONS:**

- *There are 6 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *This examination is closed book. Calculators that perform symbolic manipulations such as the TI-89, TI-92 or their equivalence, are* **not permitted**. *Other calculators may be used. Turn off and put away all* **cell phones.**

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| Question | Points | Score |
|---|---|---|
| 1 | 27 | |
| 2 | 16 | |
| 3 | 12 | |
| 4 | 14 | |
| 5 | 12 | |
| 6 | 12 | |
| 7 | 12 | |
| 8 | 18 | |
| 9 | 16 | |
| 10 | 25 | |
| 11 | 36 | |
| Total | 200 | |

1. [27 Points] Evaluate each of the following: (Credit will be given only if you show work that justifies your answer.)

a) [9 Points] $\int_0^1 (4x + e^x - 2)\, dx$. ( Decimal approximations such as 3.4567 will not receive credit.)

b) [9 Points] $\int \left(\frac{1}{2x+1} + \cos(5x-1)\right) dx$.

c) [9 Points] $\int \frac{\sqrt{\ln x}}{x}\, dx$.

2. [16 Points] Find:

a) [8 Points] $\frac{d}{dx}F(x)$, where $F(x) = \int_1^{\sqrt{x}} e^{t^4}\, dt$.

b) [8 Points] $f(x)$, if $f'(x) = \sec^2 x + \frac{1}{1+x^2}$ and $f(0) = 1$.

3. [12 Points] By using the limit definition of the derivative, find $f'(1)$ if $f(x) = \dfrac{x}{x+1}$. Other methods for finding the derivative will not receive credit. Show all your work.

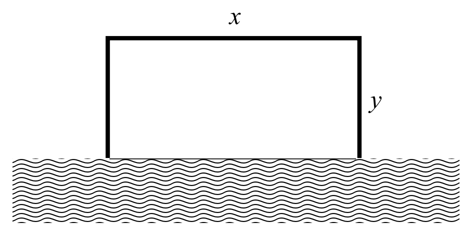
4. [14 Points] Find the equation of the tangent line to the curve $xe^{y-2} - x^4 + y^3 = 8$ at the point $(1, 2)$.

5. [12 Points] Find the **exact** value of the following limit: $\lim_{x \to 0} \dfrac{x \sin x}{1 - \cos x}$. (Show work that justifies your answer. Numerical and/or graphical reasoning is not sufficient and will receive no credit.)

6. [12 Points] A spherical balloon is being inflated with air at the rate of 6 $\text{ft}^3/\text{min}$ . How fast is the radius increasing when the radius is 5 ft? Remember, the volume enclosed by a spherical balloon of radius $r$ is $\dfrac{4\pi}{3} r^3$.

7. [12 Points] Find $\dfrac{dy}{dx}$ if $y = \dfrac{1 + \ln x}{2^x - \arcsin(5x)}$.

8. [18 Points] A farmer wants to fence a rectangular region adjacent to a straight river with area of $160\,\mathrm{m}^2$. The cost of fencing is \$5 per meter for the side parallel to the river, \$8 per meter for the sides perpendicular to the river, and no fencing is needed along the river. Find the dimensions of the rectangular region that minimizes the cost of fencing.

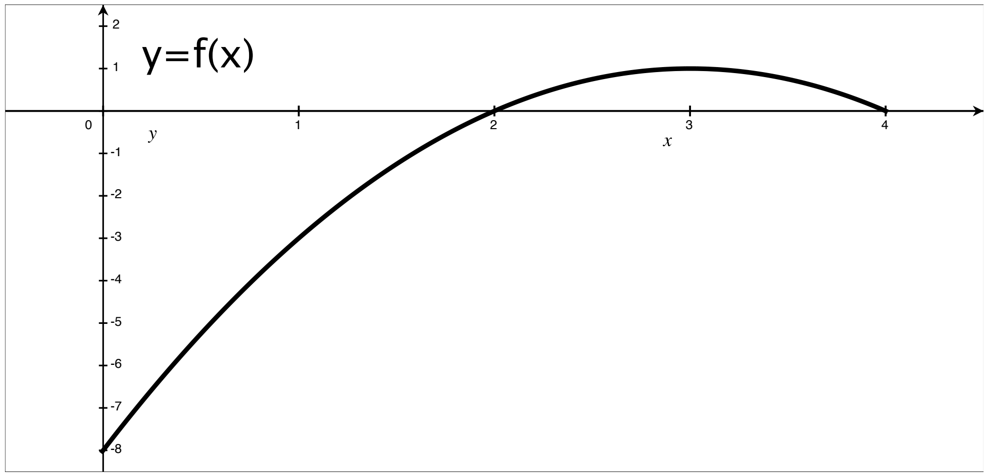



9. [16 Points] Let $y = g(x)$ be a differentiable function with $g(3) = 2$ and $g'(3) = -1$.

a) [8 Points] If $F(x) = g(\sqrt{4x+5})$, find $F'(1)$.

b) [8 Points] Find the linearization of $g$ at $x = 3$ and use it approximate $g(3.1)$.

10. [25 Points] (5 points for each part) Let $f$ be a function whose graph on the interval $[0, 4]$ is as shown below. Assume it is derivative, $f'$, is continuous on the interval $[0, 4]$. Find the **exact** value or a reasonable approximation of each of the following: (**Note:** if an exact value can be computed, then an approximation is not acceptable).




a) $f'(3)$

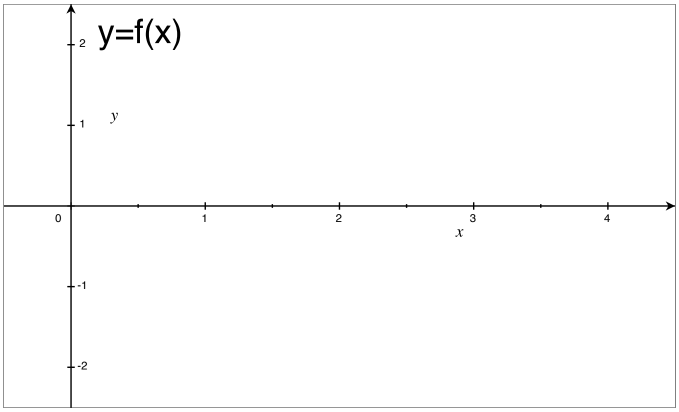
b) The average rate of change of $f$ on the interval $[0, 4]$.

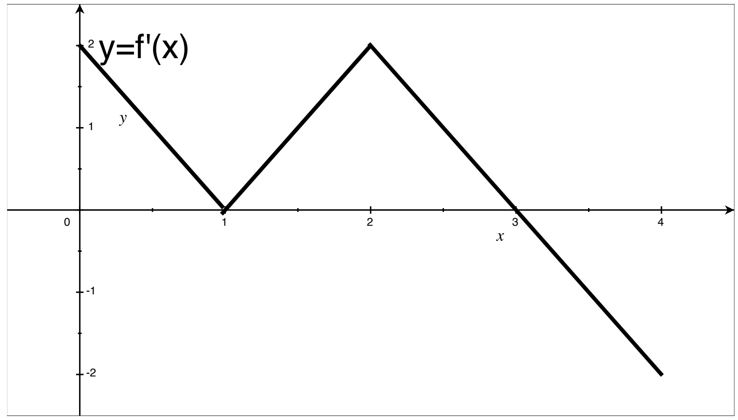
c) $\int_0^4 f'(x)dx$

d) $\int_0^4 f(x)dx$

e) The average value of $f$ on the interval $[0, 4]$.

11. [36 Points] Let $f(x)$ be a continuous function on $[0, 4]$ with $f(0) = -2$, and whose **derivative** $f'(x)$ is as shown below.




a) [10 Points] Find the $x$-coordinates of all critical points of $f$ in the interval $[0, 4]$ and classify them as local maximum, local minimum, or neither.

b) [10 Points] List all inflection points of $f$ and all intervals on which $f$ is concave up and concave down.

c) [10 Points] By using the assumption $f(0) = -2$ and the graph of $y = f'(x)$ above, find $f(3)$.

d) [6 Points] Sketch a reasonable, **but correct** graph of $y = f(x)$ in the empty plot next to the graph of $f'(x)$ on $[0, 4]$. Make sure to highlight all important features of the graph of $y = f(x)$.