Exam 3

12/5/2013 **TA Name:**_____

**You must show all of your work to receive full credit!**

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | Total |
|-----|---|---|---|---|---|---|---|-------|
| score | | | | | | | | |

(1) (8 points) Find: $\dfrac{d}{dx} \displaystyle\int_0^{\cos x} e^{t^2}\, dt$.

(2) (21 points, 7 points for each part) Evaluate the following limits (If you use L´Hôpital's rule make sure to justify its use. Note, numerical reasonings won't get any credit. If the limit is $+\infty$ or $-\infty$, say so with justification):

(a) $\displaystyle\lim_{x \to 0} \frac{\sin(7x)}{\tan(3x)}$

(b) $\displaystyle\lim_{x \to \infty} \frac{x+3}{1+\ln x}$

(c) $\displaystyle\lim_{x \to +\infty} \frac{3x+7}{e^x+4}$

(3) (12 points) A car is traveling at 60 mph due south at a point 4 miles north of an intersection. A police car is traveling at 50 mph due west at a point 3 miles east of the same intersection. How fast was the distance between the two car decreasing at that instant?
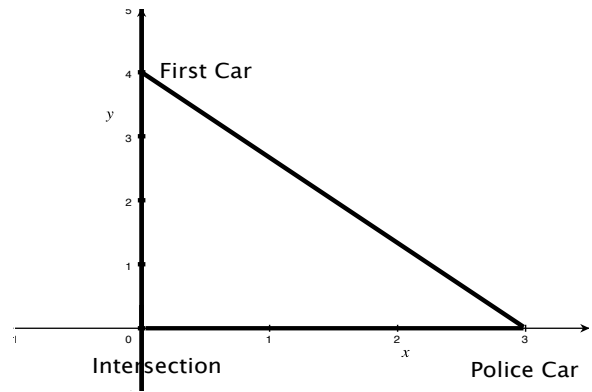


(4) (13 points) Assume that $y = f(x)$ is a continuous function on $[0, 4]$ with $f(0) = 0$ and whose derivative $y = f'(x)$ is as shown below.
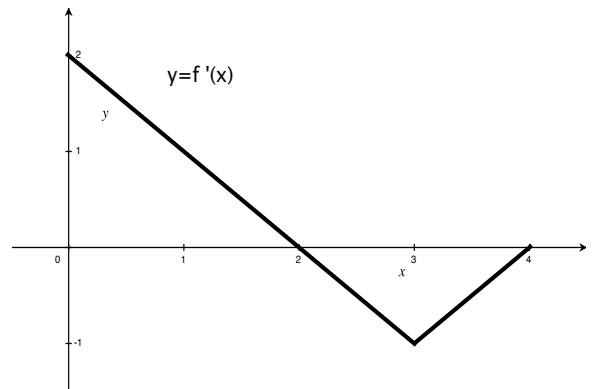


(a) (9 points) Use the Fundamental Theorem of Calculus and the graph of $y = f'(x)$ to compute $f(4)$.

(b) (4 points) Find the average value of $f'$ on $[0, 4]$.

(5) (8 points, 4 points for each part) Given the following information about two functions $f$ and $g$:

$$\int_1^7 f(x)dx = 2, \quad \int_4^7 f(x)dx = -4, \quad \int_1^7 g(x)dx = -1.$$ Find the exact value of each of the following (Make sure to show your work):

(a) $\int_1^7 (5f(x) + 4g(x))\, dx$

(b) $\int_1^4 f(x)dx$

(6) (10 points) Find, **but don't evaluate**, a definite integral whose value gives the area of the bounded region enclosed by the graphs of $y = x^2 - 4$ and $y = x + 2$. **Make sure to sketch the region.**

(7) (28 points, 7 points for each part) Evaluate the following integrals: (Decimal approximations such as 2.1234 will not get any credit.)

(a) $\int_0^1 \left(5x^4 + e^x - 1\right) dx =$

(b) $\int \left(\dfrac{3}{1+x^2} + \sin(3x+1)\right) dx =$

(c) $\int \dfrac{\sec^2 x}{3 + \tan x} \, dx =$

(d) $\int \dfrac{1}{x} \sqrt{\ln x} \, dx =$