Math 106–Section 450 (Prof. Rammaha) Your Name:_____

Exam 1 10/3/2013

TA Name:_____

Important Note: On this exam and future exams as well, your work must be shown to receive full credit. Calculators that have a built-in computer algebra system (CAS) are not permitted. Examples of such calculators include (but not all): TI-89, TI-92, TI-Nspire, HP-40, HP-41. Put away cell phones and other illegal electronic devices.

| ĺ | No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total |
|---|-------|---|---|---|---|---|---|---|---|-------|
| ĺ | Score | | | | | | | | | |

(1) (24 points) Find $\frac{dy}{dx}$ for each of the following functions (You Need Not Simplify):

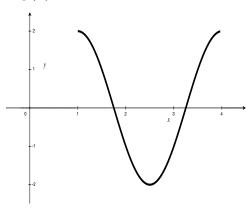
(a)(8 points)
$$y = e^x \arcsin(x) + \sin(3x) + \ln x$$
.

(b)(8 points)
$$y = \frac{2^x - \cos(4x)}{x^2 + \sec x}$$
.

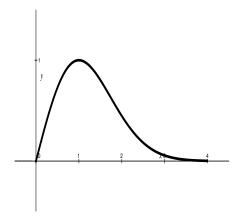
(c)(8 points)
$$y = \left(\ln(5x+4) + \tan^{-1}(\sqrt{x})\right)^{10}$$
.

(2) (10 points) By using the definition of the derivative only, find the exact value of f'(2) if $f(x) = x^2 - x$. Any other method of computing f'(2) will receive no credit.

(3) (8 points) The graph below is the graph of a sinusoidal y = g(x). Based on the graph, find the **period**, **amplitude**, and a possible formula for g(x).



(4) (10 points) Let y = f(x) be a function whose graph is as shown below. (a)(5 points) From the graph find the average rate of change of f on the interval [1,4].



(b)(5 points) On a separate set of axes, sketch a reasonable graph of y = f'(x) in the space below.

- (5) (14 points) In this problem, answers such as 2.3456 are not acceptable and will not receive any credit. So do not use your calculator and show all of your work.
 - (a)(7 points) Find all values of x that satisfy: $\log_2(x-1) + \log_2(2x+2) = 3$.

(b)(7 points) Find the exact value of: $\tan \left(\sin^{-1} \frac{2}{7} \right)$.

- (6) (12 points) Suppose y = f(x) is a differentiable function whose tangent line at x = 3 is given by 4x + y = 7.
 - (a)(6 points) Find f(3) and f'(3)
 - (b)(6 points) Let $g(x) = f(4x^2 \frac{1}{x})$. Find g'(1). Make sure to justify your answer by using the chain rule properly.

- (7) (10 points) Let $F(x) = \frac{3x+4}{x-1}$.
 - (a)(6 points) Evaluate the following limits by showing proper work:
 - $\bullet \quad \lim_{x \to \infty} F(x)$
 - $\bullet \quad \lim_{x \to 1^+} F(x)$
 - $\bullet \quad \lim_{x \to 1^-} F(x)$
 - (b)(2 points) Does F have a horizontal asymptote? If yes, what is it?
 - (c)(2 points) Does F have any vertical asymptotes? If yes, what are they?
- (8) (12 points) Let

$$G(x) = \begin{cases} x^2 - 5; & -\infty < x \le 1 \\ kx + 1; & x > 1, \end{cases}$$

where k is a constant.

(a)(8 points) Compute: $\lim_{x\to 1^+} G(x)$ and $\lim_{x\to 1^-} G(x)$.

(b)(4 points) Find the value(s) of k for which the function G is continuous on $(-\infty, \infty)$. Make sure to show all of your work to receive full credit.