FINAL EXAM REVIEW-MATH 104

1. Find $\frac{dy}{dx}$ for the following expressions y (do not simplify):

(a) $y = \frac{x^2 - 8x}{3 - 2x}$ (b) $y = e^{x^2+1}\ln(x^2 + 4)$

(c) $y = e^{x^2+1}\ln(x^2 + 4)$ (d) $y = (6x + e^{-3x})^{10}$

2. Solve the following equations for x:

(a) $3 + 2e^{4x-5} = 17$ (b) $-5 + 2\ln(3x + 4) = 1$

3. Find an equation of the tangent line to the graph of the curve $y = f(x) = (x^2 - 8)^4$ at the point $(3, 1)$.

4. Evaluate the following limits:

(a) $\lim_{x\to 1} \frac{2x^2 + 5x - 7}{3x^2 - 8x + 5}$

(b) $\lim_{x\to+\infty} \left(6 + \frac{2x^2 - 8}{3 - x^2}\right)$

5. Find all critical numbers and all inflection points on the graph of $f(x) = x^2\ln(x)$.

6. Let $y = f(x) = x^2 + 6x$.

(a) Find dy when $x = 2$ and $dx = .25$

(b) Find Δy when $x = 2$ and $\Delta x = .25$

7. Is the function $f(x) = \frac{e^x}{x}$ increasing or decreasing at $x = \frac{1}{2}$? Explain why.

8. Let $y = f(x)$ be a function such that $f''(x) = x^2(x + 4)(x - 2)$ for all $x\epsilon(-\infty, +\infty)$.

(a) List the open interval(s) where the graph of f is concave up.

(b) List the number(s) x where $(x, f(x))$ is a point of inflection on the graph of f.

9. How much money should Barbara invest on May 3, 2006 at an annual interest rate of 4.92 per cent, compounded continuously, in order to have $42,500 on May 3, 2018? (Round off your answer to the nearest cent).

10. Assume that for some commodity, the price elasticity of demand E is given by the formula $E = E(p) = \frac{2p^2}{108 - p^2}$, $0 < p < 10$ units.

Is the demand elastic or inelastic when $p = 5$? For what value of p is the revenue maximized?

11. Given the cost function $C = C(x) = x^2 + 20x + 900$ dollars, use calculus methods to determine the number of units x that should be produced in order to minimize the average cost per unit.

12. (a) Approximate the area under the graph of $y = \frac{1}{1+x^2}$ on the interval $[0, 4]$ with $n = 4$, using left endpoints.

(b) Do the same thing for $y = e^{x^2}$ on the same interval as (a) but using right endpoints.

13. Kelly invested \$12,000 in a mutual fund on May 3, 1997. On May 3, 2006 her investment was worth \$22,500.

(a) What was the annual rate of growth of this investment, assuming continuous compounding?

(b) If this mutual fund continues to appreciate at the same rate, how much will her investment be worth on May 3, 2015?

14. If a material has a half-life of 17 years, how much of a 40 gram mass will remain after 55 years? (Round off your answer to the nearest hundredth of a gram).

15. Find the following antiderivatives:
(a) $\int \left(6(x+1)^{-\frac{2}{5}} + \frac{1}{(x+2)^{10}}\right) dx$ (b) $\int x^2(2-3x^3)^{\frac{3}{2}}\, dx$

16. Evaluate the following definite integrals:

(a) $\int_0^2 \left[x^2 - e^{3x}\right] dx$
(b) $\int_1^3 \left(1 + \frac{2}{x}\right) dx$

17. If $\int_0^{10} f(x)\, dx = 12$ and $\int_4^{10} f(x)\, dx = -3$, evaluate the definite integral $\int_0^4 5f(x)\, dx$.

18. Let $p = D(q) = 40 - q^2$ dollars be the demand function and let $p = S(q) = 2q + 5$ be the supply function for some commodity.

(a) Find the equilibrium point (q_0, p_0).

(b)Find the Consumer Surplus.

19. Use the substitution method to evaluate the definite integral:

$\int_0^2 \frac{x}{\sqrt{6x^2+1}}\, dx$. Clearly identify what substitution u you are using and show all your work.

20. Use the substitution method to evaluate the definite integral:

$\int_0^3 \frac{x^2}{x^3+1}\, dx$.

21. Let R be the region enclosed by the curves $y = 5 - x^2$ and $y = x + 3$.

(a) Sketch a graph of the region R.

(b) Express the area of the region R as a definite integral. (Do not evaluate this integral.)