MATH 104 HOUR 3 REVIEW QUESTIONS

1. Use differentials to approximate:

(a) $\sqrt{51}$ (b) $\ln(.97)$ (c) $(123)^{\frac{1}{3}}$

2. Find $g''(2)$ if $g(x) = \frac{3x+1}{5x-7}$.

3. Suppose the demand equation for a commodity is $p = 350 - x^2$ dollars. Is the demand elastic or inelastic when $x = 10$. Why?

4. Use calculus methods to find the absolute maximum value M and the absolute minimum value m of the function $f(x) = 8 + 3x^2 - x^3$ on the closed interval $[1, 3]$.

5. Graph and analyze the function $f(x) = x^2 e^{-x}$, including relative extrema, concavity, and asymptotes.

6. (a) Find $g''(e)$ if $g(x) = x^2 \ln(x)$.

(b) Find all critical numbers of the function $f(x) = x^2 e^{-5x}$.

7. Assume that for some commodity, the price elasticity of demand is given by the formula $E = E(p) = \frac{5p}{144-4p}$ for $0 < p < \$36.00$. Find the price p for which the revenue is a maximum.

8. Find all points of inflection of $y = f(x) = x^4 - 4x^3 + 116$.

9. Find dy and Δy, if $y = \sqrt{x^3 - 2}$, $x = 2$, and $dx = \Delta x = 1$.

10. Given the cost function $C = C(x) = 2x^2 + 15x + 800$ dollars, use calculus methods to determine the number of units x that should be produced in order to minimize the average cost per unit.

11. Let $y = f(x)$ be a function such that $f'(x) = x^3(x-1)^2(x+4)(x-6)$ for all $x \in (-\infty, \infty)$. List the critical numbers, the open intervals on which f is increasing, and the number(s) at which f has a relative maximum.

12. (a) Use calculus-based procedures to find two positive numbers x and y such that $xy = 100$ and the function $S = x + 4y$ is a minimum.

(b) Find two numbers whose difference is 50 and whose product is a minimum.

13. Find the points of inflection of the graphs of:

(a) $f(x) = x(6-x)^2$.

(b) $f(x) = (x-2)^3(x-1)$.

14. Find an equation of the tangent line to the graph of $y = f(x) = e^{4x-3} - \ln(x^4)$ at the point $(1, e)$.

15. Suppose that a manufacturer can sell x widgets at a price of $80 - .02x$ dollars each and assume that it costs $40x + 1500$ dollars to produce all x of them.
(a) Find the revenue function, and the profit function.

(b) Determine the value of x which will maximize the revenue function.

(c) Determine the value of x that will maximize the profit function.

16. Let $y = f(x)$ be a function such that $f''(x) = x^2(x+4)(x-2)$ for all $x \epsilon (-\infty, +\infty)$.

(a) List the open interval(s) where the graph of f is concave up.

(b) List the number(s) x where $(x, f(x))$ is a point of inflection on the graph of f.

17. Find an equation of the tangent line to the graph of the curve $y = f(x) = \ln(x^3) - 6x^2$ at the point $(1, -6)$.

18. Find an equation of the tangent line to the graph of the curve $y = \frac{\ln(x)}{x}$ at the point $(e^2, 2e^{-2})$.

19. Find the differential dy for:

(a) $y = (1 - 3x^2)^3$

(b)$y = \frac{2-x}{x+5}$

20. The demand function for a certain product is modeled by $p = 60 - 0.2x$. If x changes from 6 to 7, what is the corresponding change in p? Compare the values of dp and Δp.

21. If the total profit function is modeled by $P = 0.003x^2 + 0.019x - 1200$, use differentials to approximate the change in profit corresponding to an increase in sales of one unit when $x = 750$.

22. Determine the relative extrema of the function $f(x) = x^2 - 3\ln(x)$.

23. Sketch the graph of the following functions, showing all relative extrema, points of inflection, intercepts, and asymptotes. Also, state the domain of each.

(a) $y = \frac{2}{x-3}$

(b) $y = \frac{2x}{x^2-1}$.

24. Suppose that a manufacturer wants to make an open box with a square base which is to hold 25 liters. The material used is plastic and the material for the bottom will cost three times as much as the material for the sides. What are the dimensions which will minimize the total cost?