A Topological Journey Through Spaces and Knots

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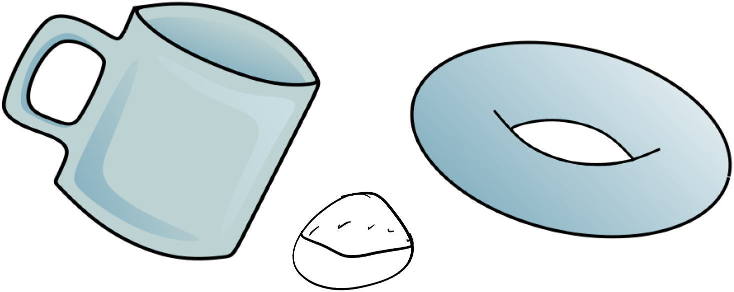
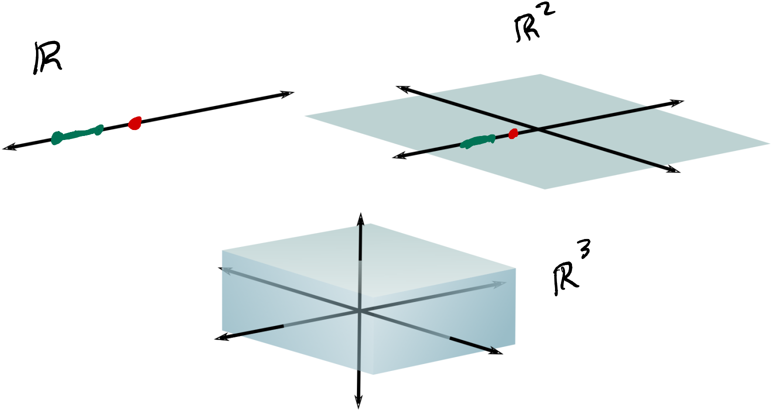
Topology

Topology is the study of spaces that can stretch and compress.

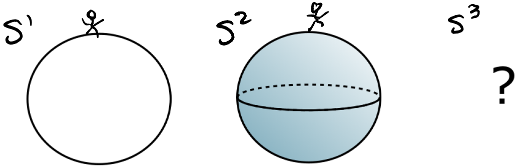


Moving Dimension to Dimension



A **manifold** is a topological space that locally looks like $\mathbb{R}^n$ for some n.

Spheres of Different Dimensions



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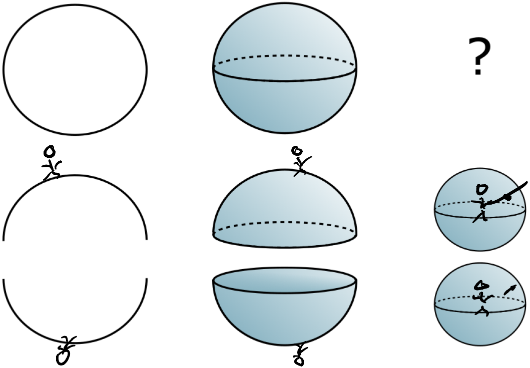

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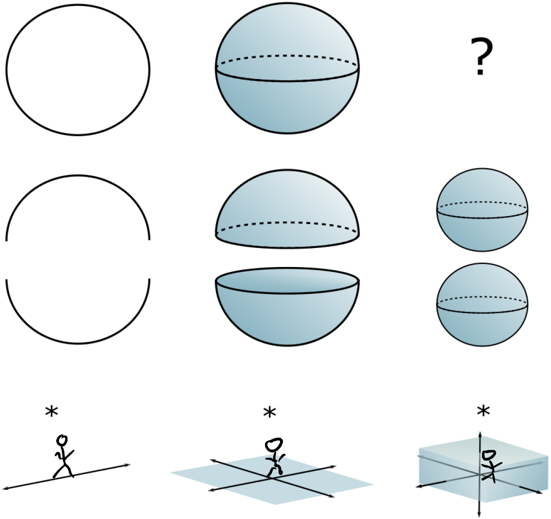
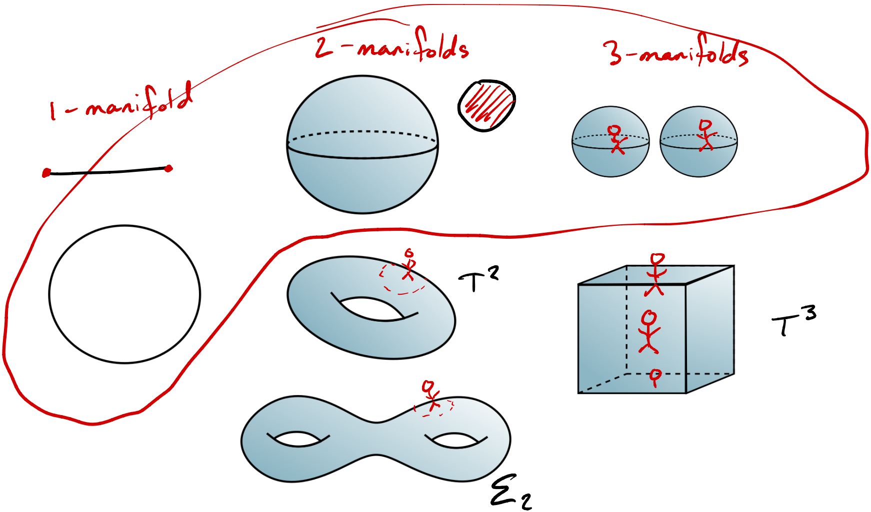
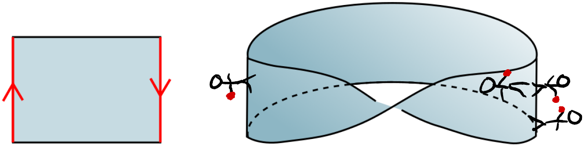


Examples of Manifolds



Non-orientable Manifolds



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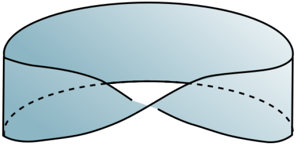
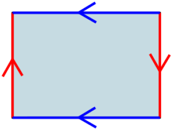
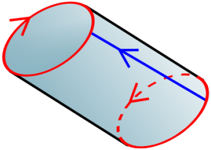
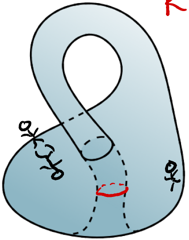




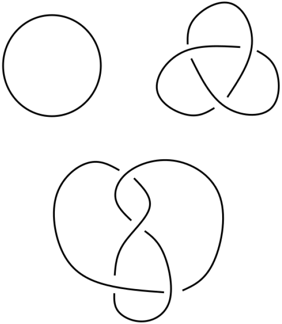




Klein Bottle needs 4-dimensions

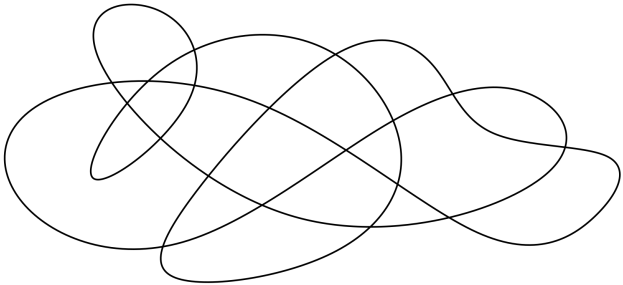
Knot Theory

In what ways can we put a circle in 3-dimensional space?



What Can We Do With Knot Projections?

Every knot projection can be checkerboard colored.



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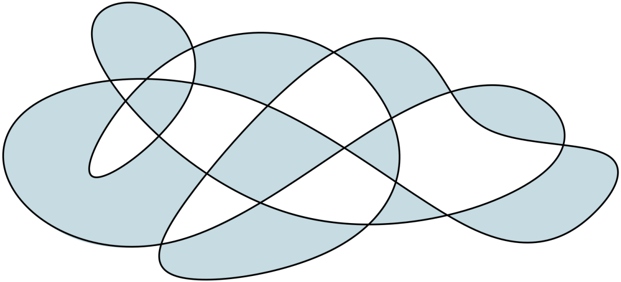
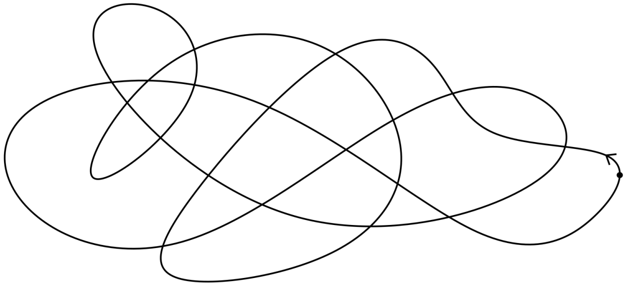


What Can We Do With Knot Projections?

The crossing information of any knot projection can be chosen to get a diagram of the unknot.



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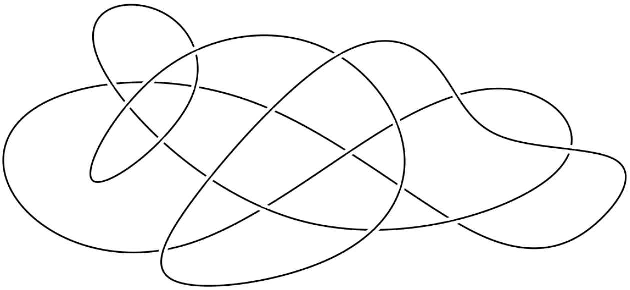
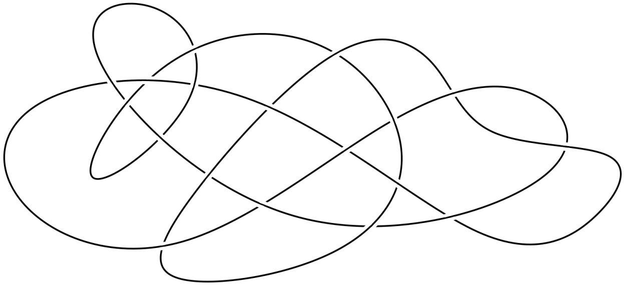


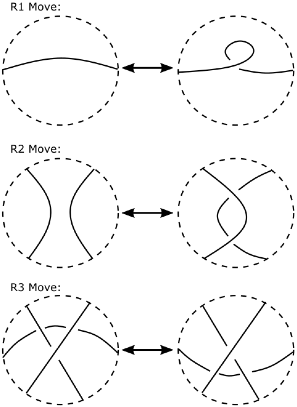
What Can We Do With Knot Projections?

The crossing information of any knot projection can be chosen to get an alternating diagram.



Reidemeister's Theorem

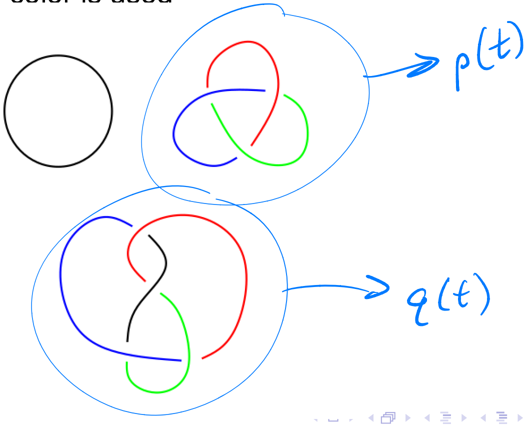
Any pair of diagrams of the same knot are related by a sequence of the Reidemeister moves below:



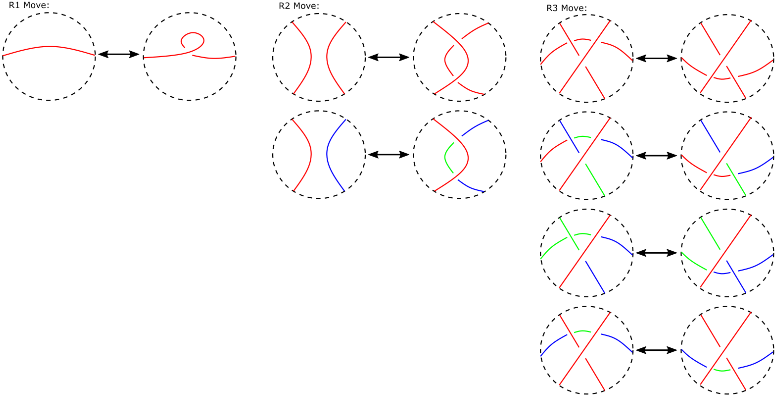
Tricolorability

A knot is **tricolorable** if the strands of its diagram can be colored with 3 colors such that:

- each crossing uses all different colors or all one color
- more than one color is used



Tricolorability is Well-Defined

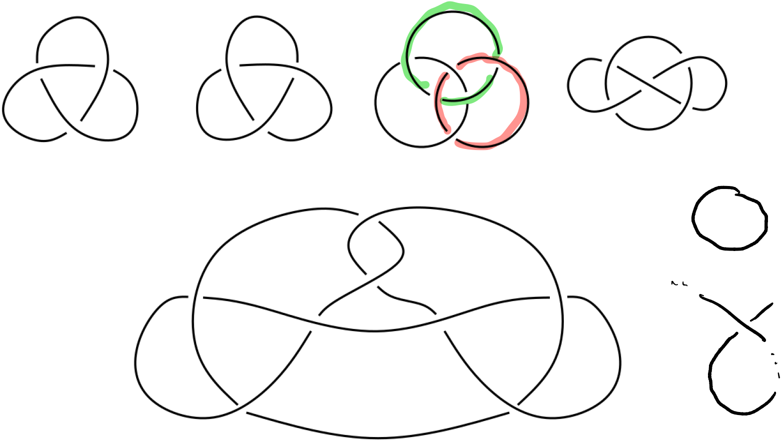


Invariants

A **knot invariant** is a property of knots which can be determined for each knot which is constant for equivalent knots.
Knot invariants:

- Tricolorability
- Unknotting number
- Crossing number
- LOTS more...

Some Fun Knots and Links



Thank you!