

Index

- α -limit set, 111
- ω -limit set, 111, 123
- Abel's formula, 195, 306
- absolute error, 163
- action integral, 249
- adjoint
 - n th-order linear, 305
 - vector equation, 302
- admissible
 - function, 233
 - variation, 240
- algebraic boundary layer function, 184
- approaches the boundary, 365
- Ascoli-Arzela theorem, 357
- asymptotically stable, 9, 57
- autonomous
 - first-order scalar, 5
 - system, 87
- averaging
 - method, 178
- basic convergence theorem, 369
- Bendixson-Dulac theorem, 129
- Bernoulli's equation, 18
- Bessel's equation, 372
- bifurcation, 14, 21
 - diagram, 15
 - pitchfork, 16, 138
 - saddle node, 14
 - transcritical, 15, 116
- big oh, 164
- boundary condition
 - linear homogeneous, 278
 - linear nonhomogeneous, 278
 - nonlinear, 278
- boundary layer, 179
- boundary layer correction, 179
- brine problem, 20
- bubble problem, 116
- calculus of variations, 240
- Cantor selection theorem, 356
- carrying capacity, 8, 12, 187
- Cauchy function
 - $(p(t)x')' = 0$, 201
 - n th-order, 284
 - example, 201
 - self-adjoint equation, 199
- Cauchy sequence, 309
- Cauchy-Peano theorem, 358
 - extended, 367
- Cayley-Hamilton theorem, 43
- center, 105
- characteristic equation, 31
- chemostat
 - example, 151
 - model, 154
- compact interval, 214
- companion matrix, 24
- comparison theorem
 - $(k, n - k)$ conjugate BVP, 295
 - for BVP, 264
 - self-adjoint equations, 217
- competition model, 154, 159
- completing the square lemma, 233
- continuity
 - with respect to initial conditions, 372
 - with respect to parameters, 372
- contraction mapping
 - definition, 309
- contraction mapping theorem, 309
- coupled vibrations, 25
- cycle, 120
 - limit, 120
- degenerate node, 99, 101
- disconjugate
 - n th-order, 294
 - criterion, 265
 - definition, 212

- divergence
 - vector field, 129
- dominant solution, 224
- double zero, 212
- double-well oscillator, 160
- Duffing equation, 189
- eigenfunction, 205
- eigenpair, 205
 - definition, 31
- eigenvalue
 - definition, 31
 - simple, 205
 - SLP, 205
- eigenvector
 - definition, 31
- epidemic
 - example, 151
 - model, 154
- equicontinuous, 356
- equilibrium point, 89
 - definition, 6
 - isolated, 6
- error
 - absolute, 163
 - relative, 163
- Euler-Lagrange equation, 243
- existence and uniqueness theorem
 - self-adjoint equation, 193
 - linear vector equation, 25
 - scalar case, 2
- existence-uniqueness
 - matrix case, 37
- existence-uniqueness theorem
 - $(k, n - k)$ BVP, 287
 - $(k, n - k)$ right-focal BVP, 295
 - n th-order linear equations, 281
 - autonomous system, 88
- exponential boundary layer function, 184
- extendability theorem, 393
- extension, 363
- extension theorem, 365
- fast variable, 181
- first variation, 241
- Fisher's equation, 168
 - with convection, 189
- Fite-Wintner theorem, 232
- fixed point, 309
- Floquet
 - theorem, 69
 - multiplier, 70
 - system, 65
 - theory, 65
- Floquet exponent, 86
- Floquet multiplier
 - example, 70
- forced pendulum problem, 341
- formal adjoint
 - vector equation, 302
- formally self-adjoint
 - scalar n th-order, 305
 - vector equation, 302
- fundamental lemma of the calculus of variations, 243
- fundamental matrix
 - definition, 40
 - theorem, 41
- gene activation, 21
- general solution, 29
- generalized Gronwall's inequality, 392
- generalized logistic equation, 11
- glycolysis, 158
 - model, 156
- gradient, 109
- gradient system, 111, 127, 153
- Green's formula, 196
- Green's function, 253, 259, 260, 263
 - $(k, n - k)$ BVP, 289
 - jump condition, 279
- Green's function
 - General Two Point BVP, 253
 - Periodic BVP, 269
- Green's theorem, 306
- Gronwall's inequality, 392
 - generalized, 392
- Hamilton's principle, 249
- Hamiltonian function, 94
- Hamiltonian system, 93
- heteroclinic orbit, 93
- Hill's equation, 74
- homoclinic orbits, 92
- Hopf bifurcation, 133
 - example, 158
- hystereses, 16
- index
 - of a curve, 128
- initial point, 2
- initial value, 2
- inner product, 195
 - complex-valued functions, 306
 - weight function, 206
- integral equation, 347
- integral means, 372
- Jacobi equation, 244

- Jacobian matrix, 348
 Jordan canonical form, 65
 Keplerian model, 173
 Kneser's theorem, 375, 399
 Lagrange bracket, 195
 Lagrange identity, 195, 305
 Lagrangian, 249
 LaSalle Invariance Theorem, 112
 LaSalle invariance theorem
 problem, 153, 154
 left maximal interval of existence, 365
 Legendre's equation, 372
 Legendre's necessary condition, 245
 problems, 277
 Leibniz rule, 202
 Liapunov function, 109
 problem, 152
 strict, 109
 Liapunov's Inequality, 265
 Liapunov's stability theorem, 109
 limit cycle, 120
 limit cycles
 example, 156
 linear differential equation
 first-order, 4
 linear operator
 definition, 26
 linear space
 definition, 26
 linear system, 23
 linear vector equation, 23
 linearly dependent
 vector functions, 28
 vectors, 27
 Liouville's formula, 282
 Liouville's theorem, 37
 nth-order scalar, 282
 Lipschitz condition
 definition, 348
 local section, 122
 logistic
 equation, 8
 logistic equation, 18
 generalized, 11
 logistic growth, 8, 87
 with harvesting, 21
 Lorenz system, 159
 lower solution, 326
 Lozinski measure, 61
 Mammmana factorization, 228
Mathematica, 145
 Mathieu's equation, 74, 175, 190
 matrix equation
 first order, 36
 matrix exponential
 properties, 49
 matrix log
 example, 67
 theorem, 66
 matrix norm, 349
 max norm, 313
 maximal interval of existence, 2, 88, 365
 maximum solution, 387
 method of averaging, 178
 method of multiple scales, 178
 minimum solution, 387
 Riccati equation, 236
 model
 chemostat, 154
 competition, 154, 159
 epidemic, 154
 glycolysis, 156
 multiple scales
 method, 178
 Nagumo condition, 334
 Newton's law of cooling, 5, 19
 Newton's law of motion, 1
 nondimensional, 164
 nonextendability
 example, 3
 nonoscillatory, 213
 nonsingular
 matrix, 40
 nonuniqueness
 example, 3
 norm
 Euclidean, 56
 max, 56, 313
 traffic, 56
 vector case, 56
 normalized solutions, 292
 normed linear space, 309
 nullclines, 95
 orbit, 7, 88
 order of differential equation, 1
 ordinary differential equation, 1
 orthogonal
 weight function, 206
 oscillatory, 213
 pendulum equation, 93, 278
 pendulum problem, 92, 189
 friction, 112, 117, 152
 per capita growth rate, 8
 period doubling, 159

- route to chaos, 140
- periodic boundary conditions, 275
- periodic solution, 122, 155, 172
- perturbation
 - methods, 161
 - series, 161
- perturbations
 - singular, 178
- phase line diagram, 7
- phase plane diagram, 90
- Picard iterate
 - exercise, 397
- Picard iterates, 310, 350
- Picard-Lindelof theorem, 350
- Picone identity
 - scalar case, 217
- pitchfork bifurcation, 16, 138
- plankton example, 328
- Poincaré-Bendixson
 - example, 123
 - theorem, 122
- Poincaré-Bendixson theorem
 - example, 156
- Polya factorization, 219
 - n*th-order, 298
- positive definite
 - quadratic functional, 234
- positively invariant, 111
- potential energy function, 9
- predator-prey
 - example, 116, 125, 154, 159
- principal solutions, 301
- proper global minimum, 241
- Putzer algorithm
 - example, 46
 - example, 45, 47
 - theorem, 43
- quadratic functional, 233
- Rössler
 - attractor, 160
 - system, 159
- radioactive decay, 1
- recessive solution, 224
- reduction of order
 - exercise, 276
- relative error, 163
- renormalization, 174
- Riccati equation, 229
- Riccati operator, 229
- Riccati substitution, 230
- right maximal interval of existence, 365
- Routh-Hurwitz criterion, 134
- saddle point, 103, 117
- saddle-node bifurcation, 14
- saturation level, 12
- second variation, 241
- self-adjoint
 - scalar *n*th-order, 305
 - vector equation, 302
- self-adjoint equation, 192
- semigroup property, 108
- sensitive dependence on initial conditions, 140
- separation of variables, 207
- separatrix, 92
- separatrix
 - problem, 149
- shock layer, 184
- shooting method, 326
- similar matrices, 77
- simple zero, 212
- simply connected domain
 - definition, 129
- singular perturbation, 178
- singular Sturm-Liouville problem, 275
- skew Hermitian, 302
- solution, 1
 - initial value problem, 2
 - integral equation, 347
 - vector equation, 345
 - vector IVP, 347
- species example, 151
- spiral point, 106
 - stable, 107
 - unstable, 108
- spring problem
 - coupled masses, 25
- spurious, 186
- stability theorem
 - constant matrix case, 57
 - stable, 9, 57
 - asymptotically, 9, 57
 - globally asymptotically, 57
 - limit cycle, 120
 - unstable, 9
- stable manifold, 117
- stable node, 98
- stable spiral point, 106
- stability theorem, 113
- strange attractor, 140
- strict Liapunov function, 109
- Sturm comparison theorem, 217
- Sturm separation theorem, 213
- Sturm-Liouville, differential equation, 204
- Taylor's theorem, 307

terminal velocity, 20, 179
trace of matrix
 definition, 37
trajectory, 88
transcritical bifurcation, 15, 116
transient chaos, 139
traveling wave, 169, 170, 188
Trench factorization, 223
 *n*th-order, 301
trivial solution
 vector equation, 31
uniform Lipschitz condition
 definition, 348
 exercise, 396
uniformly bounded
 definition, 356
uniqueness theorem, 368
unitary matrix, 308
unstable, 9, 57
unstable manifold, 117
unstable node, 97
unstable spiral point, 106
upper solution, 326

van der Pol, 158
van der Pol equation, 132
van der Pol's equation, 189
variation of constants formula
 example for vector case, 52
 vector case, 51
first-order scalar, 4
 higher order, 285
self-adjoint equation, 202
variational equation, 380, 383
Velhurst
 equation, 8

Weierstrass integral formula, 247
Wronskian, 195
Wronskian determinant
 k functions, 282