

*Errata for Discrete Fractional Calculus as of May 25, 2018:*

**CHAPTER 1**

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- (1) Page 17, line 16 “ $+\frac{f(t)}{1-p(t)+q(t)}$ ”
- (2) Page 24, line 13 replace “1.44” by “1.35”
- (3) Page 24, line 15 replace second “cos” by “sin”
- (4) Page 26, line 3- replace “ $\cosh_p(t, a)$ ” by “ $\cos_p(t, a)$ ”
- (5) Page 26, line 2- replace “ $\sinh_p(t, a)$ ” by “ $\sin_p(t, a)$ ”
- (6) Page 26, line 1- “ $n > k \geq 0$ ”
- (7) Page 28, line 6- “ $c_3 = \frac{1}{2}$ ”
- (8) Page 28, line 5- replace last “2” by “ $\frac{1}{2}$ ”
- (9) Page 28, line 1- replace second “d” by “d-1”
- (10) Page 29, line 6- replace “b” by “b+1”
- (11) Page 30, line 5- replace “derivative” by “difference”
- (12) Page 41, line 7 replace “(1-t)” by “(t-1)”
- (13) Page 44, line 1- replace first “t” by “t<sup>2</sup>”
- (14) Page 49, line 1

$$e_{\frac{\alpha}{t}}(t, a) \cos_{\frac{\beta}{1+\alpha}}(t, a), \quad e_{\frac{\alpha}{t}}(t, a) \sin_{\frac{\beta}{1+\alpha}}(t, a)$$

- (15) Page 49, line 9- should be “ $(t\Delta - \alpha_1)(t\Delta - \alpha_2) \cdots (t\Delta - \alpha_n)y = 0$ ”
- (16) Page 50, line 1 replace “ $\mathbb{R}_n$ ” by “ $\mathbb{R}^n$ ”
- (17) Page 56, line 6 replace “i” by “k”
- (18) Page 59, line 3- omit this line
- (19) Page 65, line 15- replace “ $\lambda$ ” by “ $\lambda_0$ ”
- (20) Page 68, line 8- replace “ $(\sqrt{3})^t$ ” by “ $c(\sqrt{3})^t$ ” Also replace “ $(\sqrt{3})^{t+1}$ ” by “ $c(\sqrt{3})^{t+1}$ ”

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**CHAPTER 2**

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- (1) Page 101, line 6 This formula is incorrect

- (2) Page 102, line 6 should be “ $= -\frac{\nu\Gamma(t-s)}{\Gamma(\nu+1)\Gamma(t-s-\nu+1)}$ ”
- (3) Page 106, line 2- replace “ $\mathbb{N}_0$ ” by “ $\mathbb{N}_{\nu-N}$ ”
- (4) Page 107, line 2 replace “ $\mathbb{N}_0$ ” by “ $\mathbb{N}_{\nu-N}$ ”
- (5) Page 107, line 4 replace “ $\mathbb{N}_0$ ” by “ $\mathbb{N}_{\nu-N}$ ”
- (6) Page 119, line 4 replace “ $\Delta_a^{k+1}f(t)$ ” by “ $\Delta_a^{-(k+1)}f(t)$ ”
- (7) Page 119, line 7- replace “ $h_{\nu-1}$ ” by “ $h_{-\nu-1}$ ”
- (8) Page 120, line 4 replace “ $\Delta_a^{1+\nu}f(t)$ ” by “ $\Delta_a^\nu f(t)$ ”
- (9) Page 120, line 6- “ $\int_a^t \Delta f(\tau)$ ” by “ $\int_a^t \Delta f(\tau)\Delta\tau$ ”
- (10) Page 121, line 8- replace “ $f(t)$ ” by “ $f(\tau)$ ”
- (11) Page 121, line 6-  $\int_a^{t-\nu+1} h_{\nu-2}(t, \sigma(\tau))f(\tau)\Delta\tau$
- (12) Page 122, line 2  $h_{\nu-k+j}(t, a)$
- (13) Page 122, line 3  $\Delta_a^{k-\nu+1}(t, a)$
- (14) Page 122, line 4 omit “ $\Delta_a^{-\nu}$ ”
- (15) Page 136, line 5 replace “(2.38)” by “(2.39)”
- (16) Page 141, line 8 “ $a - N - \nu$ ”

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### **CHAPTER 3**

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- (1) Page 152, line 12 The last sentence on this line should appear right after Definition 3.4
- (2) Page 156, line 10- omit second “:”
- (3) Page 158, line 5- replace lower limit to “ $t+1$ ” and upper limit to “ $t$ ”
- (4) Page 160, line 5 and 6 it should be “ $1 - p(\tau)$ ”
- (5) Page 161, line 7 it should be “ $\alpha \square (\beta \square p)$ ”
- (6) Page 163, line 5- replace “ $g$ ” by “ $q$ ”
- (7) Page 164, line 7 omit “where  $t_0 \in \mathbb{N}_{a+1}$  and  $A, B \in \mathbb{R}$ ”
- (8) Page 173, line 12  $\int_a^t E_p(s, a)\nabla s = \frac{1}{p}E_p(s, a)|_a^t$
- (9) Page 174, line 4- At the beginning of this line put an equal sign
- (10) Page 174, line 2- replace “ $\frac{1}{4}$ ” by “ $\frac{3}{4}$ ”
- (11) Page 177, line 4 lower limit of integration is  $a$

- (12) Page 177, line 7- replace “ $\ominus 3$ ” by “ $\boxminus 3$ ”
- (13) Page 178, lines 12,13,14 replace “ $\ominus 3$ ” by “ $\boxminus 3$ ”
- (14) Page 183, line 3- replace “ $\mathbb{N}_0$ ” by “ $\mathbb{N}_{a+1}$ ”
- (15) Page 186, line 13- replace “(iii)” by “(iv)”
- (16) Page 187, line 5- replace “ $\rho(t)$ ” by “ $\sigma(t)$ ”
- (17) Page 187, line 4- need  $f(\tau)$  in this sum
- (18) Page 187, line 2- at the end of this line add “ $+H_{\nu-1}(t, \rho(a))f(a)$ ”
- (19) Page 189, line 15 need an “ $f(\tau)$ ” under the integral sign
- (20) Page 190, lines 1,6,8,9,10,4-,2- replace upper limit of sum by  $N + m - 1$
- (21) Page 190, line 1 replace in two places “M” by “m”
- (22) Page 190, lines 6, 7, 4-, 3- replace “M” by “m”
- (23) Page 191, lines 1, 2 replace upper limit of sum by  $N + m - 1$
- (24) Page 196, line 6- omit “ $,|s|^p < 1$ ”
- (25) Page 202, line 1 replace “a+1” by “a”
- (26) Page 204, line 9, 15 replace “r” by “ $\min\{1, r\}$ ”
- (27) Page 204, line 11- omit “ $s^\nu$ ”
- (28) Page 204, line 9- replace “1” by “ $\min\{1, r\}$ ”
- (29) Page 204 In Lemma 3.88 we assume  $|s - 1| < r$
- (30) Page 205 In Theorem 3.89 we assume  $f$  is of exponential order  $r > 0$  and  $|s - 1| < \min\{1, r\}$
- (31) Page 207, line 10- replace “nonnegative” by “not negative”
- (32) Page 208, line 9 the first “ $-\nu$ ” should be “ $\nu$ ”
- (33) Page 208, line 13 replace “3.89” by “3.90”
- (34) Page 208, line 14  $RHS = s^\nu \mathcal{L}_{a+1}\{H_\mu(\cdot, a)\}(s) + \frac{s^\nu - s^{N-1}}{1-s} H_\mu(a+1, a) - \sum_{k=2}^N s^{N-k} [\nabla^{k-1} H_\nu(t, a)]_{t=a+1}$
- (35) Page 208, line 15 should be  $= s^\nu \mathcal{L}\{H_\mu(\cdot, a)\}(s) + \frac{s^\nu - s^{N-1}}{1-s} - \sum_{k=2}^N s^{N-k}$
- (36) Page 209, line 2 should be (3.42)
- (37) Page 209, line 8 replace “ $h(t)$ ” by “ $f(t)$ ”
- (38) Page 209 lines 10,12,16 replace the lower limit “a” by “ $a + 1$ ”
- (39) Page 209, line 10- replace “ $f(a+)$ ” by “ $f(a + N + 1)$ ”

- (40) Page 209, At the end of the proof of Theorem 3.94 add the following: Hence we see that the solution of the given IVP is uniguely determined at  $t = a + N + 1$ . In a similar manner if we let  $t = a + N + 2$  in equation (3.43) we can show that the solution of the IVP is uniguely determined at  $a + N + 2$ . Using mathematical induction we obtain the desired result that the solution of the given IVP is uninguely determined on  $\mathbb{N}_{a+1}$ .
- (41) Page 210, line 8- replace “ $\rho(a)$ ” by “ $\sigma(a)$ ”
- (42) Page 210, line 7- replace “ $\rho(a)$ ” by “ $\sigma(a)$ ”
- (43) Page 214, line 8 replace “ $2k + 1$ ” by “ $2k$ ”
- (44) Page 215, In Theorem 3.103 we assume  $\alpha k + \beta + 1$  is not a negative integer for  $k = 0, 1, 2, \dots$ .
- (45) Page 217, line 8- should be “ $\nabla_{\rho(a)}^{-\nu} f(t)$ ”
- (46) Page 220, line 7 “ $H_{-\nu-1}(t, \rho(\tau))f(\tau)\nabla\tau$ ”
- (47) Page 221, line 6- replace “(3.85)” by “Theorem 3.85”
- (48) Page 222, line 9- need “ $\nabla\tau$ ” at the end of this line
- (49) Page 222, line 8- replace last term by  $\int_{a+k}^t H_{\nu-2}(t, \rho(\tau))\nabla^{k-1}f(\tau)\nabla\tau$
- (50) Page 222, line 7- replace this line by  $= -\nabla^{k-1}f(a+k)H_{\nu-1}(t, a+k)+H_{\nu-2}(t, \tau)\nabla^{k-2}f(\tau)|_{a+k}^t + \int_{a+k}^t H_{\nu-3}(t, \rho(\tau))\nabla^{k-2}f(\tau)\nabla\tau$
- (51) Page 222, line 6- replace this line by  $= -\nabla^{k-1}f(a+k)H_{\nu-1}(t, a+k) - H_{\nu-2}(t, a+k)\nabla^{k-2}f(a+k) + \int_{a+k}^t H_{\nu-3}(t, \rho(\tau))\nabla^{k-2}f(\tau)\nabla\tau$ .
- (52) Page 222, lines 5-, 4- omit these two lines
- (53) Page 223, line 7, replace “ $H_{N-\nu-k+j}(t, a)$ ” by “ $\nabla^N H_{N-\nu-k+j}(t, a+k)$ ” note 2 changes
- (54) Page 223, line 8 omit this line
- (55) Page 223, line 9 replace last “a” by “a+k”
- (56) Page 223, lines 2-,3- in both of these 2 lines replace “h” by“H”
- (57) Page 224, line 8- in 2 places replace “h” by“H”
- (58) Page 224, replace the end of this line by “the given IVP.”
- (59) Page 225, line 2, 3, 4-, 1- in each of these 4 lines replace “h” by“H”
- (60) Page 225, line 4 replace “the equation.” by “equation  $\nabla_a^\nu x(t) = g(t)$ ” to get that
- (61) Page 226, lines 2-4 in 5 places replace “h” by “H”

- (62) Page 233, line 10 there is a missing parenthesis
- (63) Page 234, line 14 need a minus sign in front of the summation
- (64) Page 234, line 14 replace “ $f(a + i - 1)$ ” by “ $f(a + i)$ ”
- (65) Page 237, line 6- replace “ $(-\mu + k + N - i)$ ” by “ $(-\mu + k + N - i - 1)$ ”
- (66) Page 237, line 2- replace “ $\rho(a)$ ” by “ $a$ ”
- (67) Page 237, line 1- replace first “ $a$ ” by “ $a + 1$ ”
- (68) Page 267, In Theorem 3.175 we assume  $0 < \nu < 1$
- (69) Page 268, line 1 replace “that” by for “ $t \geq a + 1$ ”
- (70) Page 268, line 2  $RHS = \nabla[\nabla_{a^*}^\nu \nabla_a^{-\nu}(1)](t + 1)$
- (71) Page 268, line 3  $RHS = \nabla[\nabla_a^{-(N-\nu)} \nabla^N \nabla_a^{-\nu}(1)](t + 1)$
- (72) Page 268, line 4  $RHS = [\nabla(1)](t + 1)$

## **CHAPTER 4**

- (1) Page 290, line 1-  $(t - s)\frac{n}{q}$
- (2) Page 291, line 7-  $D_q h_{n+1}(\alpha, t)$
- (3) Page 292, line 13 replace every “ $\alpha$ ” that appears as an exponent including falling exponents that have an “ $\alpha$ ” in them by “ $\nu$ ”
- (4) Page 292, line 14 replace second “ $\alpha$ ” by “ $\nu$ ”
- (5) Page 293, throughout this page replace every “ $\alpha$ ” that appears as an exponent including falling exponents that have an “ $\alpha$ ” in them by “ $\nu$ ”
- (6) Page 293, line 3 replace “ $t \geq s$ ” by “ $t \leq s$ ”
- (7) Page 295, line 1 replace “ $q^\alpha$ ” by “ $q^\nu$ ”
- (8) Page 307, line 12- replace “ $f(t)$ ” by “ $F(t)$ ”
- (9) Page 337, line 1- replace “ $[n]_p!$ ” by “ $[n - 1]_p!$ ”
- (10) Page 339, line 2, 7- replace “ $\alpha + 1$ ” by “ $\alpha - 1$ ”
- (11) Page 342, line 6- replace “ $q^k$ ” by “ $q^i$ ”
- (12) Page 345, line 2 replace “ $(p - 1)$ ” by “ $(1 - p)$ ”
- (13) Page 345, line 3 replace first minus sign with a plus sign

(14) Page 345, line 3 replace last plus sign with a minus sign

(15) Page 346, line 6 replace last "t" by "s"

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**CHAPTER 5**

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**CHAPTER 6**

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**CHAPTER 7**

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