# Preface

This book provides an introduction both to real analysis and to a range of important applications that depend on this material. Three-fifths of the book is a series of essentially independent chapters covering topics from Fourier series and polynomial approximation to discrete dynamical systems and convex optimization. Studying these applications can, we believe, both improve understanding of real analysis and prepare for more intensive work in each topic. There is enough material to allow a choice of applications and to support courses at a variety of levels.

This book is a substantial revision of *Real Analysis with Real Applications*, which was published in 2001 by Prentice Hall. The major change in this version is a greater emphasis on the latter part of the book, focussed on applications. A few of these chapters would make a good second course in real analysis through the optic of one or more applied areas. Any single chapter can be used for a senior seminar.

The first part of the book contains the core results of a first course in real analysis. This background is essential to understanding the applications. In particular, the notions of limit and approximation are two sides of the same coin, and this interplay is central to the whole book. Several topics not needed for the applications are not included in the book but are available online, at both this book's official website `www.springer.com/978-0-387-98097-3` and our own personal websites, `www.math.uwaterloo.ca/~krdavids/` and `www.math.unl.edu/~adonsig1/`.

The applications have been chosen from both classical and modern topics of interest in applied mathematics and related fields. Our goal is to discuss the theoretical underpinnings of these applied areas, showing the role of the fundamental principles of analysis. This is not a methods course, although some familiarity with the computational or methods-oriented aspects of these topics may help the student appreciate how the topics are developed. In each application, we have attempted to get to a number of substantial results, and to show how these results depend on the theory.

This book began in 1984 when the first author wrote a short set of course notes (120 pages) for a real analysis class at the University of Waterloo designed for students who came primarily from applied math and computer science. The idea was to get to the basic results of analysis quickly, and then illustrate their role in a variety of applications. At that time, the applications were limited to polynomial approximation, Newton's method, differential equations, and Fourier series.

A plan evolved to expand these notes into a textbook suitable for a one- or two-semester course. We expanded both the theoretical section and the choice of applications in order to make the text more flexible. As a consequence, the text is not uniformly difficult. The material is arranged by topic, and generally each chapter gets more difficult as one progresses through it. The instructor can omit some more difficult topics in the early chapters if they will not be needed later.

We emphasize the role of normed vector spaces in analysis, since they provide a natural framework for most of the applications. So some knowledge of linear algebra is needed. Of course, the reader also should have a reasonable working knowledge of differential and integral calculus. While multivariable calculus is an asset because of the increased level of sophistication and the incorporation of linear algebra, it is not essential. Some of this background material is outlined in the review chapter.

By and large, the various applications are independent of each other. However, there are references to material in other chapters. For example, in the wavelets chapter (Chapter 15), it seems essential to make comparisons with the classical approximation results for Fourier series and for polynomials.

It is possible to use an application chapter on its own for a student seminar or topics course. We have included several modern topics of interest in addition to the classical subjects of applied mathematics. The chapter on discrete dynamical systems (Chapter 11) introduces the notions of chaos and fractals and develops a number of examples. The chapter on wavelets (Chapter 15) illustrates the ideas with the Haar wavelet. It continues with a construction of wavelets of compact support, and gives a complete treatment of a somewhat easier continuous wavelet. In the final chapter (Chapter 16), we study convex optimization and convex programming. Both of these latter chapters require more linear algebra than the others.

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We welcome comments on this book.

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## Part B Applications