1. Consider the straight line given by $f(x) = mx + b$ where $m \neq 0$.

   (a) Give a convincing (algebraic) argument of why $f$ is a one-to-one function. Your argument does not need to be long, but it does need to use the definition of one-to-one.

   (b) Find a formula for the inverse of $f$.

   (c) If the graphs of two functions are parallel lines with a nonzero slope, what can you say about the graphs of the inverses of the functions?

   (d) If the graphs of two functions are perpendicular lines, each with a nonzero slope, then what can you say about the graphs of the inverses of the functions?

   *Solution.* For (a), the definition says that, for a function $f(x)$ to one-to-one, if $f(x_1) = f(x_2)$, then we must have $x_1 = x_2$. So suppose $f(x_1) = f(x_2)$, that is, by the definition of $f(x)$,

$$mx_1 + b = mx_2 + b$$

   Subtracting $b$ from both sides gives

$$mx_1 = mx_2$$

   Since $m \neq 0$, we can divide both sides by $m$ to get

$$x_1 = x_2.$$

   This shows that if $f(x_1) = f(x_2)$, we must have $x_1 = x_2$, that is, the only way two $x$-values can produce the same value of $f(x)$ is if the two $x$-values are equal.
   For (b), we switch $x$ and $y$ and solve for $y$. First,

$$x = my + b$$

   Subtracting $b$ from both sides gives

$$x - b = my$$

   Since $m \neq 0$, we can divide both sides by $m$ to get

$$\frac{x-b}{m} = y$$

   Thus, $y = x/m - b/m$. So the formula fo $f^{-1}(x)$ is

$$f^{-1}(x) = \frac{x}{m} - \frac{b}{m}.$$

   That is, the inverse function is a line with slope $1/m$ and $y$-intercept $-b/m$.
   For (c), notice that parallel lines have the same slope. Call this slope $m$. Then the inverse functions of these two lines will both have slope $1/m$ from part (b). So the graphs of the inverses of the functions will also be parallel lines.
   For (d), if the two lines are perpendicular, then one will have slope $m$ and the other will have slope $-1/m$. The corresponding inverse functions will be lines with slopes $1/m$ and $1/(-1/m) = -m$. If we take the negative of the reciprocal of $1/m$, we get $-m$, so the two inverse functions are perpendicular lines, just like the ones we started with.