Math106sp08 **Exam #3** **23 Apr 08**

Name ____

Please Circle your Recitation Section:

9:30	151-Katie	152-Yanqui	153-Anatoly	154-Nathan	155-Joe
11:30	251-Katie	252-Charlie	253-Travis	254-Anatoly	255-Nathan
1:30	351-Joe	352-Yanqui	353-Anne		
6:30	101-Ahlschwede				

Read the questions carefully and answer them fully. Show all your work. No symbolic algebra calculators may be used. All calculus work must be shown to receive credit. You have 1.5 hours for this 100 point exam.

Q# (pts)	**1** (12)	**2** (16)	**3** (20)	**4** (16)	**5** (12)	**6** (12)	**7** (12)	Total (100)
Score								

1] Consider the region bounded by y = x + 3 and y = x^2 – x + 3. Sketch this region and find its area.

graph of $x+3$ 2pts

graph of x^2-x+3 2pts

$x^2 - x + \cancel{3} = x + \cancel{3}$

$x^2 - 2x = 0$

$x(x-2) = 0$

1 2

$$\int_0^2 (x+3) - (x^2 - x + 3)\,dx$$

1

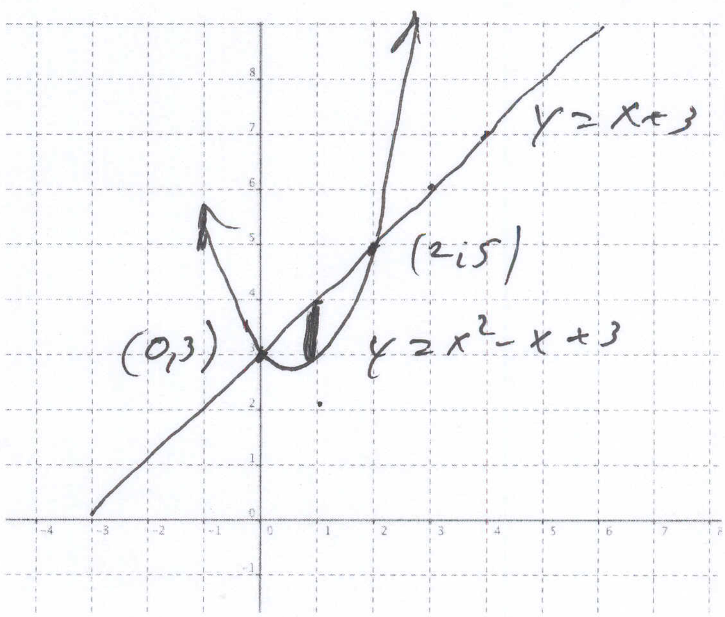
$$= \int_0^2 x + 3 - x^2 + x - 3\,dx$$

$$= \int_0^2 -x^2 + 2x\,dx$$

$$= -\frac{x^3}{3} + x^2 \Big|_0^2 \quad 2$$

$$= \left(-\frac{2^3}{3} + 2^2\right) - 0$$

$$= 4 - \frac{8}{3} = \frac{4}{3} \quad 2$$



2] Function values of a continuous differentiable function are shown in the table below.

x	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5	7
f(x)	6	7	8	8	11	12	14	17	20	23	27	31	37

Use this information to find values for:

A] $\int_1^7 f(x)dx$ using a left hand Riemann sum with 6 divisions.

B] $\int_1^7 f(x)dx$ using a midpoint Riemann sum with 3 divisions.

C] Consider now that f(x) is actually the velocity of a particle moving along a wire in feet per second. With correct units, describe what your answer to A] means. Include reasoning as to if your value is too large or too small.

6 → A. $f(1)(2-1) + f(2)(3-2) + f(3)(4-3) + f(4)(5-4)$

5 → $+ f(5)(6-5) + f(6)(7-6)$

$= 6(1) + 8(1) + 11(1) + 14(1) + 20(1) + 27(1)$

1 $= 86$

6 → B note: $x_0 =$ [illegible], $x_1 = 3$, $x_2 = 5$, $x_3 = 7$

$f\left(\frac{3+1}{2}\right)(3-1) + f\left(\frac{5+3}{2}\right)(5-3) + f\left(\frac{7+5}{2}\right)(7-5)$

5 → or $f(2)2 + f(4)\cdot 2 + f(6)\cdot 2$

$= 8(2) + 14(2) + 27(2)$

1 $= 98$

4 → C Since the rectangles are now time × rate

2 we are collecting distance (feet). So A says I have travel 86 feet from time $t =$ [illegible] to $t = 7$.

Since the velocity is positive the distance

2 function is increasing, therefore the L.H.S will be less than the actual answer.

3] Evaluate the following. Answers without supporting justification and formulas will not be accepted.

5

A] $\sum_{i=1}^{30} 2i^2 - 3i + 2$

$\sum_{i=1}^{30} 2i^2 - 3i + 2$

$2\sum_{i=1}^{30} i^2 - 3\sum_{i=1}^{30} i + \sum_{i=1}^{30} 2$

$2\left(\frac{30\cdot 31\cdot 61}{6}\right) - 3\left(\frac{30\cdot 31}{2}\right) + 2\cdot 30$

$\frac{56730}{3} - \frac{2790}{2} + 60$

$= 18910 - 1395 + 60$

$= 17575$ (1)

5

B] $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{2i}{n}\right)^2 - \frac{4i}{n}\right]$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n}\left[\left(\frac{2i}{n}\right)^2 - \left(\frac{4i}{n}\right)\right]$

$\lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{2}{n}\frac{4i^2}{n^2} - \frac{2}{n}\left(\frac{4i}{n}\right)\right]$

$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8}{n^3} i^2 - \frac{8}{n^2} i$

$= \lim_{n\to\infty} \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{8}{n^2}\left(\frac{n(n+1)}{2}\right)$

$= \lim_{n\to\infty} \frac{8}{3} + \frac{6}{n} + \frac{8}{n^2} - 4 - \frac{4}{n}$

$= \frac{8}{3} - 4 = -\frac{4}{3}$ (1)

7

C] $\int_1^4 \frac{1}{\sqrt{x}} - x^3\,dx$

$\int_1^4 \frac{1}{\sqrt{x}} - x^3\,dx$

$x^{-1/2}$

$2x^{1/2} - \frac{x^4}{4}\Big|_1^4$ (4)

$= \left(2(\sqrt{4}) - \frac{4^4}{4}\right) - \left(2\sqrt{1} - \frac{1}{4}\right)$

$= (4 - 64) - (2 - \frac{1}{4})$ (2)

$= -60 - \frac{7}{4}$

$= -61\frac{3}{4}$ (1)

3

D] $\int \sec(2x)\tan(2x)dx$

$= \frac{1}{2}\sec 2x + C$

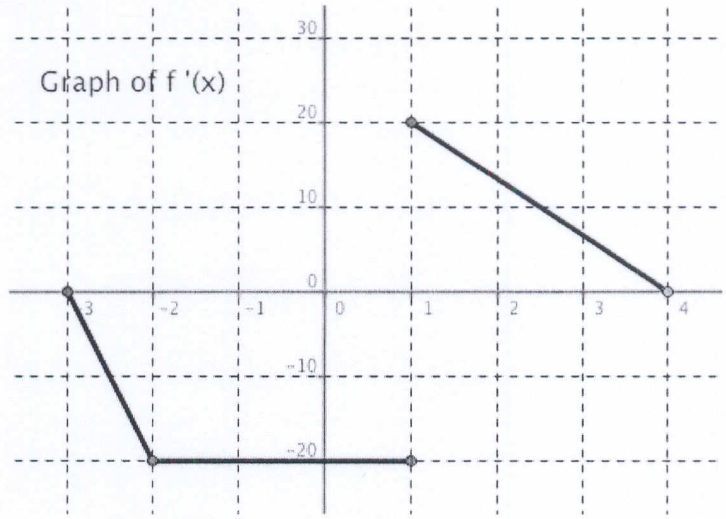
(2) (1)

4] The graph of **f '(x)** is shown at right. f(x) is continuous. Use the graph and f(0) = 6 to answer the following questions.

A] What is f(4)?

B] What is f(-3)?

C] Is f(x) concave up or down at x = 3? Explain.



6 A. $f(0) = 6$

$f(1) = 6 - 1\cdot 20 = -14$

$f(4) = -14 + \frac{1}{2}(20)(4-1)$

$= -14 + 30$

$= 16$

6 B. $f(0) = 6$

$f(-3) = +\left(\frac{2+3}{2}\right)(20) + 6$

$= +56$

4 C. $f''(x)$ is the rate of change of $f'(x)$ at $x=3$ f' is decreasing, therefore $f''(x)$ is negative so $f(x)$ is concave down at $x=3$ 2

2

5] Determine:

7 A] $\int_{-1}^{1}(4x+4)\left(x^2+2x\right)^5 dx$

2 $u = x^2 + 2x$

$du = (2x+2)dx$

$= 2(x+1)dx$

1 $\int_{-1}^{1} 2[2(x+1)](x^2+2x)^5 dx$ 1

$= 2\int_{-1}^{1} u^5 du = 2\frac{u^6}{6}\Big|_{-1}^{1}$

$= \frac{1}{3}(x^2+2x)^6\Big|_{-1}^{1}$ 2

$= \frac{1}{3}\left[\left((1)^2+2\cdot 1\right)^6 - \left((-1)^2+2(-1)\right)^6\right]$

$= \frac{1}{3}\left(3^6 - (-1)^6\right) = 3^5 - \frac{1}{3} = \frac{728}{3}$ 1

5 B $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx = 2\int e^u du$ 1

2 $u = (x)^{1/2}$

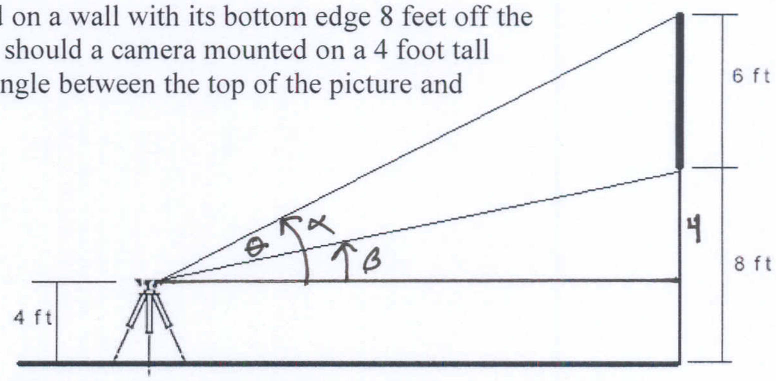
$du = \frac{1}{2\sqrt{x}}dx$

$= 2e^u + c$

$= 2e^{\sqrt{x}} + C$

2

6] A picture that is 6 feet tall is mounted on a wall with its bottom edge 8 feet off the ground. How far away from the picture should a camera mounted on a 4 foot tall tripod be placed so as to maximize the angle between the top of the picture and the bottom of the picture?



Notice: $\tan\theta = \tan(\alpha - \beta)$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$$

Now use your knowledge of the tangent function to solve the problem.

$\boxed{\text{6. solution}}$ see problem for hint

Look for $\tan\alpha, \tan\beta$

$$\tan\theta = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{\frac{10}{x} - \frac{4}{x}}{1+\frac{10}{x}\frac{4}{x}}$$

$$= \frac{6x}{x^2+40} \quad 4$$

$$\sec^2\theta \frac{d\theta}{dx} = \frac{(x^2+40)6 - 6x(2x)}{(x^2+40)^2}$$

$$= \frac{6x^2+240-12x^2}{(x^2+40)^2}$$

$$= \frac{-6x^2+240}{(x^2+40)^2} \quad 4$$

$$\sec^2\theta \frac{d\theta}{dx} = 0 \to -6x^2+240 = 0$$

$$\to x^2 = 40$$

$$x = \sqrt{40}$$

$$x = 2\sqrt{10} \quad 2$$

$$\theta = \alpha - \beta = \arctan\frac{x}{10} - \arctan\frac{x}{4} \quad 2$$

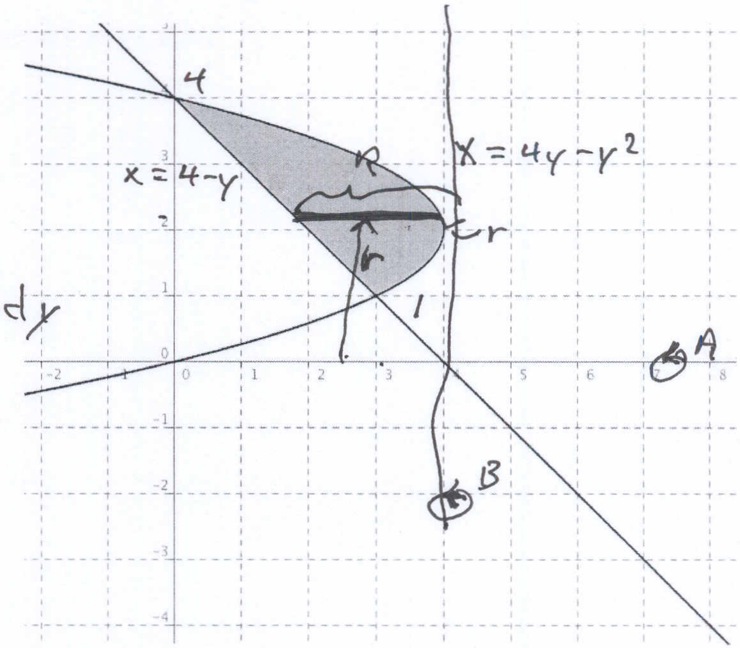
$$\frac{d\theta}{dx} = \frac{1}{1+\left(\frac{x}{10}\right)^2}\cdot\frac{1}{10} - \frac{1}{1+\left(\frac{x}{4}\right)^2}\cdot\frac{1}{4} = \frac{10}{x^2+100} - \frac{4}{16+x^2} \quad 4$$

$$= \frac{10(16+x^2) - 4(x^2+100)}{(x^2+100)(x^2+16)} = \frac{10x^2 - 4x^2 \ldots}{(\ldots)} = \frac{6x^2 - 240}{(x^2+100)(x^2+16)}$$

as before $x = 2\sqrt{10}$ 2

7] Consider the region R bounded by $x + y = 4$ and $x = 4y - y^2$ shown. Setup but **DO NOT EVALUATE** the volume obtained when the region R is revolved about:

A] The x-axis, using 1 integral
B] The line $x = 4$, using 1 integral



$\boxed{6}$ A. shell method

$$\int_1^4 2\pi(y-0)\left[(4y-y^2)-(4-y)\right]dy$$

or

$$\int_1^4 2\pi y\,(5y-y^2-4)\,dy$$

$\boxed{6}$ B.

$$\int_1^4 \pi\left[4-(4y-y^2)\right]^2 - \pi\left[4-(4-y)\right]^2 dy$$

or

$$\pi\int_1^4 (4-4y+y^2)^2 - y^2\,dy$$