Math106sp08 Exam #2 26 Mar 08

Name: ____________________

Circle your Recitation Section:

9:30	151-Katie	152-Yanqui	153-Anatoly	154-Nathan	155-Joe
11:30	251-Katie	252-Charlie	253-Travis	254-Anatoly	255-Nathan
1:30	351-Joe	352-Yanqui	353-Anne		
6:30	Ahlschwede				

Read the questions carefully and answer them fully. Show all your work. No symbolic algebra calculators may be used. You have 1.5 hours for this 100-point exam.

Q# (pts)	1 (12)	2 (12)	3 (34)	4 (7)	5 (17)	6 (8)	7 (10)	Total
Score								

(12) 1. If $x = 2t - t^3$ and $y = 1 + t^3$,

a. find $\frac{dy}{dx}$. $= \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}} = \dfrac{3t^2}{2-3t^2}$

6 ② ② ②

b. find the value(s) of t for which the slope of the tangent line to the curve is 2.

6

$\dfrac{3t^2}{2-3t^2} = \dfrac{2}{1}$ ②

$3t^2 = 4 - 6t^2$

$9t^2 = 4$

$t^2 = \frac{4}{9}$

$t = \pm \frac{2}{3}$ ④

(12) 2. a. Find the linear approximation for $f(x) = \sqrt[3]{x}$ near 8.

8

$f'(x) = \dfrac{1}{3x^{2/3}}$ ①

$L(x) = f(8) + f'(8)(x-8)$

$L(x) = 2 + \frac{1}{12}(x-8)$ or $L(x) = \frac{4}{3} + \frac{1}{12}x$

① ② ② ②

b. Use your approximation to approximate $\sqrt[3]{7.92}$.

4

$L(7.92) = \left(2 + \frac{1}{12}(7.92 - 8)\right) = 2 + \frac{1}{12}(-.08)$

plugging: 2pts

$= 2 - .00\overline{6} = 1.99\overline{3}$

ans: 2pts

(24) 3. Given $f(x) = 3x^4 - 4x^3 - 6x^2 + 6$,

9 a. Find and classify the critical x-values of f(x).

(3) $f'(x) = 12x^3 - 12x^2 - 12x = 0$ $\quad 12x(x^2 - x - 1) = 0$

$x = 0 \quad x = \frac{1 \pm \sqrt{1-4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$

f': $-$ $+$ $-$ $+$; $\frac{1-\sqrt{5}}{2}$, 0, $\frac{1+\sqrt{5}}{2}$

(2) $x = \frac{1-\sqrt{5}}{2}$ and $\frac{1+\sqrt{5}}{2}$ ~~abs~~ mins (2)

loc mins

(2) $x = 0$ loc max

7 b. Find all inflection points.

(3) $f''(x) = 36x^2 - 24x - 12 = 0$ $\quad 12(3x+1)(x-1) = 0$ (2)

$x = -\frac{1}{3}$ or 1 $\quad f''$: $+$ $-$ $+$; $-\frac{1}{3}$, 1

$\left(-\frac{1}{3}, \frac{149}{27}\right)$ (2) and $(1, -1)$ are inflection pts.

$\frac{149}{27} \approx 5.52$

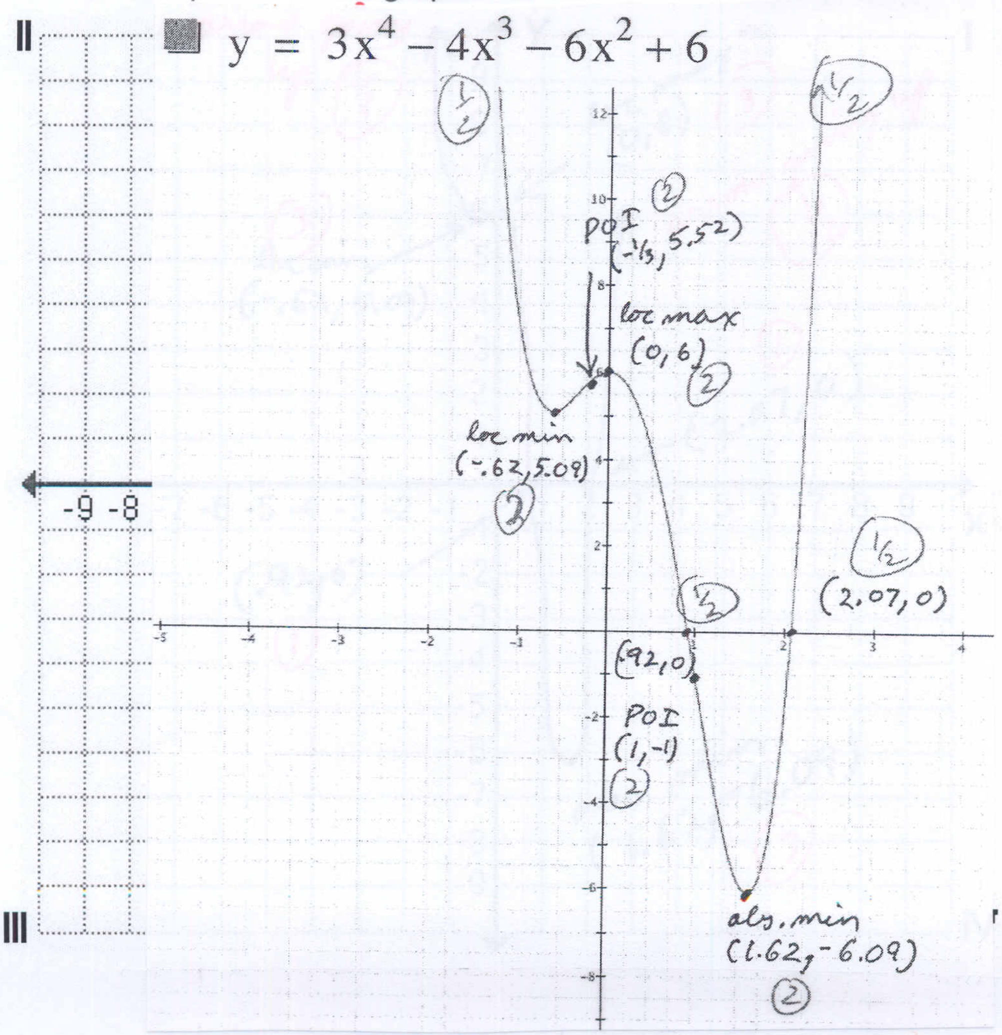
4 c. List the interval(s) on which f(x) is increasing.

(2) $\left(\frac{1-\sqrt{5}}{2}, 0\right) \cup \left(\frac{1+\sqrt{5}}{2}, \infty\right)$ (2)

2 d. List the interval(s) on which f(x) is concave down.

$\left(-\frac{1}{3}, 1\right)$ (2)

12 e. Graph f(x) on the grid below, labeling all intercepts, asymptotes, max/mins, and inflection points on the graph.



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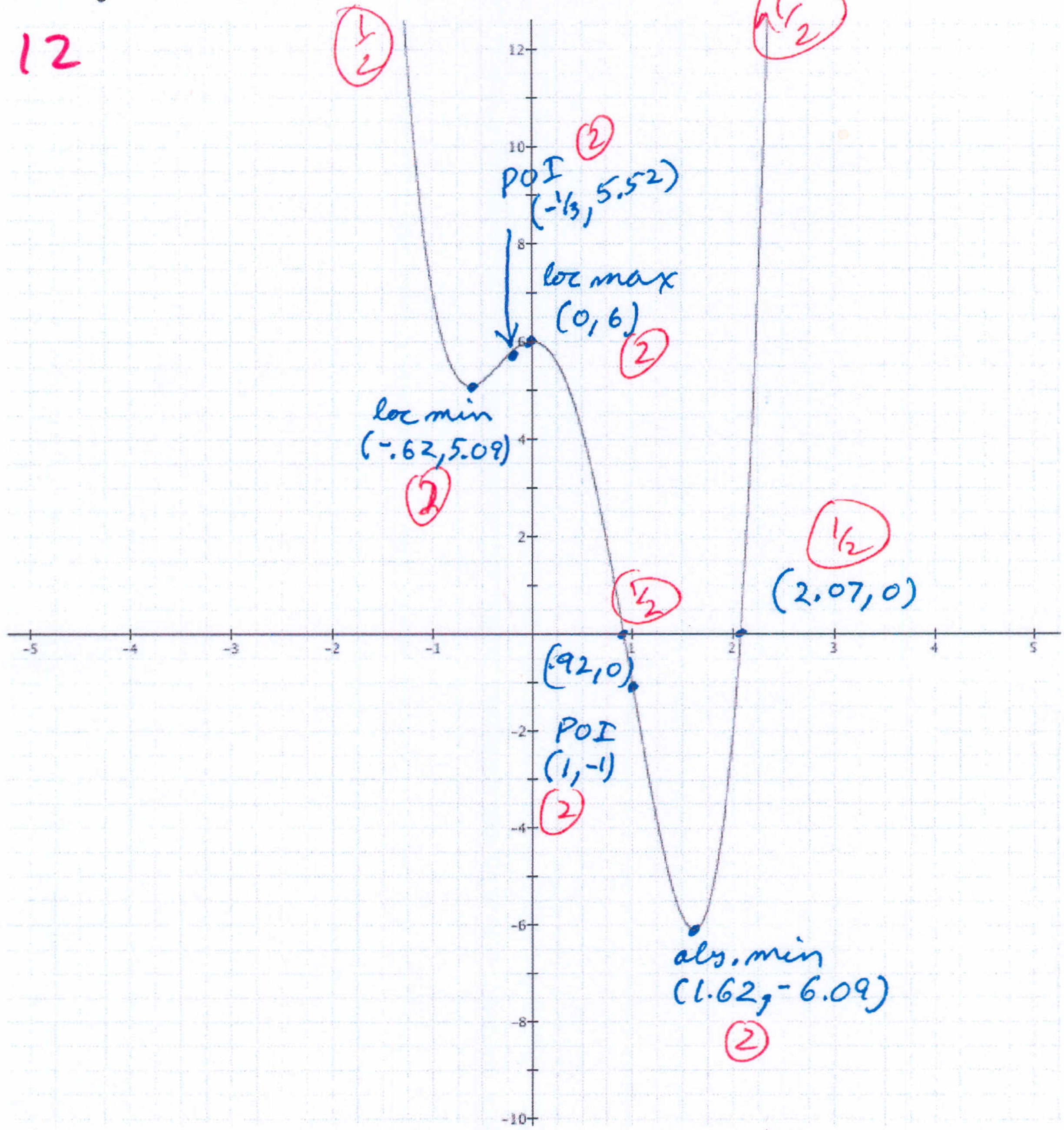
■ $y = 3x^4 - 4x^3 - 6x^2 + 6$

12



) 4. Find the following limit (be sure to show your work): $\lim_{x\to\infty} \frac{(\ln x)^2}{x^2}$ $\frac{\infty}{\infty}$ ← 1

$$\lim_{x\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{\ln x}{x^2} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2}$$

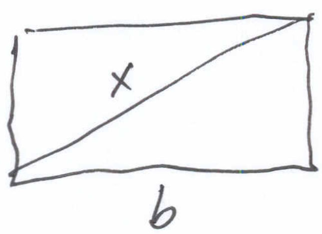
$= 0$ 1

if the drop the lim early $x\to\infty$ → −1 ←

17) 5. A rectangle's base, b, is increasing at a rate of 3 cm/min while it's height, h, is decreasing at a rate of 1 cm/min. At the time when b = 60 and h = 25

a. how is the area changing?

8



$\frac{dh}{dt} = -1$

$\frac{db}{dt} = 3$

1 $A = bh$

1 $\frac{dA}{dt} = b\frac{dh}{dt} + h\frac{db}{dt}$ 4

$= 60(-1) + 25(3)$

$= 15\ cm^2/min$ 2

b. how is the length of the diagonal changing?

9

$x^2 = b^2 + h^2$

$\cancel{2}x\frac{dx}{dt} = \cancel{2}b\frac{db}{dt} + \cancel{2}h\frac{dh}{dt}$ 2 2 2

$\underset{b}{60^2} + \underset{h}{25^2} = \underset{x}{65^2}$

$x = 65$ 1

$65\frac{dx}{dt} = 60(3) + 25(-1)$

$\frac{dx}{dt} = \frac{31}{13} \approx 2.38\ cm/min.$ 2

(8) 6. a. Using Newton's Method, write out the equation for x_{n+1} when $f(x)=3x^2-\sqrt{10}$.

6

$$x_{n+1} = x_n - \frac{3x_n^2-\sqrt{10}}{6x_n} = \frac{6x_n^2-3x_n^2+\sqrt{10}}{6x_n} = \frac{3x_n^2+\sqrt{10}}{6x_n}$$

2 4

or

2 b. f(x) has a root near x = 1, find approximations for the values of x_1 *and* x_2 to 6 decimal places. You do not need to show your work.

1 $x_1 = 1.027046$

1 $x_2 = 1.026690$

0) 7. Given $f(x)=\sin x$ defined on the interval $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$:

4 a. Verify the hypotheses of the Mean Value Theorem for f(x).

2 — f is cont on $[-\pi/2, \pi/2]$ (sin x is everywhere cont)

2 — f is diff on $(-\pi/2, \pi/2)$ (cos x is cont there and has no corners or cusps)

b. Find the value or values of c that satisfy the equation $\frac{f(b)-f(a)}{b-a}=f'(c)$ in the

6 conclusion of the Mean Value Theorem.

2 $$\frac{\sin(\pi/2)-\sin(-\pi/2)}{\pi/2-(-\pi/2)} = \cos c \quad \text{where } c \in (-\pi/2, \pi/2)$$

2 $$\frac{2}{\pi} = \cos c$$

$c \approx .880689$ and $-.880689$ 2

need both