

Important Formulas for Test 3:

1. **Antiderivatives:**

<u>Function</u>	<u>Antiderivative</u>	<u>Function</u>	<u>Antiderivative</u>
1. x^n	$\frac{1}{n+1}x^{n+1} + C$	2. $\sin kx$	$-\frac{1}{k}\cos kx + C$
3. $\cos kx$	$\frac{1}{k}\sin kx + C$	4. $\sec^2 kx$	$\frac{1}{k}\tan kx + C$
5. $\csc^2 kx$	$-\frac{1}{k}\cot kx + C$	6. $\sec kx \tan kx$	$\frac{1}{k}\sec kx + C$
7. $\csc kx \cot kx$	$-\frac{1}{k}\csc kx + C$	8. e^{kx}	$\frac{1}{k}e^{kx} + C$
9. $\frac{1}{\sqrt{1-k^2x^2}}$	$\frac{1}{k}\arcsin kx + C$	10. $\frac{1}{1+k^2x^2}$	$\frac{1}{k}\arctan kx + C$
11. $\frac{1}{x}$	$\ln x + C, \quad x \neq 0$		
12. a^{kx}	$\left(\frac{1}{k \ln a}\right)a^{kx} + C, \quad a \geq 0, a \neq 1$		

2. **Summation:**

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{2}, \quad \sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

3. **Riemann Sum** of $\int_a^b f(x)dx$:

Right Hand Rule: $\frac{b-a}{n} \sum_{k=1}^n f\left(a + k\left(\frac{b-a}{n}\right)\right)$

Left Hand Rule: $\frac{b-a}{n} \sum_{k=1}^n f\left(a + (k-1)\left(\frac{b-a}{n}\right)\right)$

4. **Integration Rules:**

Order of Integration: $\int_b^a f(x)dx = -\int_a^b f(x)dx$ Zero Width: $\int_a^a f(x) = 0$

Constant Multiple: $\int_a^b kf(x)dx = k \int_a^b f(x)dx$

Additivity: $\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$

Summation:
$$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

5. **Area under a curve** $f(x)$ for x on $[a, b]$: $A = \int_a^b f(x)dx$

Area between the curves $f(x)$ and $g(x)$ for x on $[a, b]$: $A = \int_a^b (f(x) - g(x))dx$

6. **Fundamental Theorem of Calculus:** If f is continuous on $[a, b]$ then $F(x) = \int_a^x f(t)dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$:

F.T.C.1:
$$F'(x) = \frac{d}{dx} \int_a^x f(t)dt = f(x) \quad \text{and}$$

F.T.C.2:
$$\int_a^b f(x)dx = F(b) - F(a)$$

7. Substitution Rules:

Indefinite Integrals: If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I , then

$$\int f(g(x)) \cdot g'(x)dx = \int f(u)du$$

Definite Integrals: If g' is continuous on the interval $[a, b]$ and f is continuous on the range of g , then

$$\int_a^b f(g(x)) \cdot g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

8. Volumes by Slicing and Rotation About an Axis:

Disc or Washer Method: For a solid of revolution about the x -axis, if the cross-sectional area is $A(x)$, where $R(x)$ is the outer radius and $r(x)$ is the inner radius, the volume is

$$V = \int_a^b A(x)dx = \int_a^b \pi ([R(x)]^2 - [r(x)]^2) dx$$

Cylindrical Shell Method: For a solid generated by revolving the region between the x -axis and the graph of $y = f(x) \geq 0$ with $L \leq a \leq x \leq b$ about a vertical line $x = L$, the volume is given by

$$V = \int_a^b 2\pi(x - L)f(x)dx$$

Use the Cylindrical Shell Method for solids of revolution about the y -axis that are difficult to do with the washer method.