

- 5 1. Solve the initial value problem:

$$\frac{d^2y}{dx^2} = 5x, y'(0) = 8, y(0) = 2$$

Solution. We evaluate

$$y' = \int 5x dx = \frac{5}{2}x^2 + c_1$$

Then we apply the initial condition $y'(0) = 8$ and get

$$y'(0) = \frac{5}{2}(0)^2 + c_1 = c_1 = 8$$

Hence

$$y' = \frac{5}{2}x^2 + 8$$

Next, we evaluate

$$y = \int \frac{5}{2}x^2 dx = \frac{5}{6}x^3 + 8x + c_2$$

Then we apply the other initial condition $y(0) = 2$ and get

$$y(0) = \frac{5}{6}(0)^3 + 8(0) + c_2 = c_2 = 2$$

Therefore,

$$y = \frac{5}{6}x^3 + 8x + 2$$

- 5 2. Use finite approximation to estimate the area under the graph of the following function using left-hand rule with four rectangles of equal width.

$$f(x) = x^2 + 1, -2 \leq x \leq 2$$

Solution. We divide $[-2,2]$ into 4 subintervals and each subinterval has the length 1. Then we apply the left-hand rule

$$\begin{aligned} A &\approx f(-2) \cdot 1 + f(-1) \cdot 1 + f(0) \cdot 1 + f(1) \cdot 1 = ((-2)^2 + 1) + ((-1)^2 + 1) + ((0)^2 + 1) + ((1)^2 + 1) \\ &= 10 \end{aligned}$$