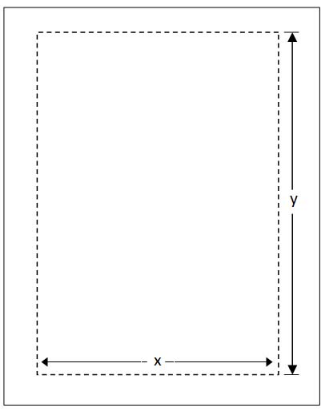
MATH 106-150
Calculus I

Quiz #8

Name: ____________________

The majority of the credit you receive will be based on the completeness and the clarity of your responses. Show your work, and avoid saying things that are untrue, ambiguous, or nonsensical. This quiz has 1 questions, for a total of 10 points.

(10 points) 1. You are designing a rectangular poster to contain 48 in.^{2} of printing with a 1-in. margin at the top and bottom and a 3-in. margin at each side. What values of x and y will minimize the amount of paper used?



(1) $xy = 48$

The total width of the page with the margins is $x+6$ and the total height is $y+2$. So the total amount of paper used is:

(2) minimize $A = (x+6)(y+2)$

We can substitute $y = \dfrac{48}{x}$ from equation (1) into equation (2), so we get equation (2) in one variable: $A = (x+6)(\dfrac{48}{x}+2)$

$$A = 48 + 2x + \frac{288}{x} + 12 = 60 + 2x + \frac{288}{x}$$

$$A' = 2 - \frac{288}{x^2} = 0 \quad \Rightarrow \quad 2 = \frac{288}{x^2} \quad \Rightarrow \quad 2x^2 = 288 \quad \Rightarrow \quad x^2 = 144 \quad \Rightarrow \quad x = \pm 12$$

But $x \geq 0$, so $x = -12$ is not a possible solution.

Now, to show $x = 12$ gives a minimum for A, we use the second derivative test:

$$A'' = \frac{576}{x^3} \quad \Rightarrow \quad A''(12) = \frac{576}{1728} > 0$$

Since A is concave up at $x = 12$, this must be a minimum.

Finally, when $x = 12$, $y = \dfrac{48}{12} = 4$.