Math 106 **Name:** ______________

Quiz 4 Answer Key **21 February 2008**

Show all work to receive full credit.

1. If $f(x) = \cos(e^{-x^2})$, find $f'(x)$.

Solution:

$$\begin{aligned} f'(x) &= \frac{d}{dx}\cos(e^{-x^2}) \\ &= -\sin(e^{-x^2})\frac{d}{dx}e^{-x^2} \\ &= -\sin(e^{-x^2})(e^{-x^2})\frac{d}{dx}-x^2 \\ &= -\sin(e^{-x^2})(e^{-x^2})(-2x) \end{aligned}$$

2. Consider the curve given by the parametric equations

$$\begin{aligned} x &= 2t^3 + 1 \\ y &= t^4 \\ -5 \leq &\, t \leq 5 \end{aligned}$$

Find $\frac{d^2y}{dx^2}$ when $t = -1$.

Solution: First, we find the first derivative.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{6t^2} = \frac{2t}{3}$$

Next, we compute the second derivative.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\frac{2t}{3}}{\frac{d}{dt}(2t^3+1)} = \frac{\frac{2}{3}}{6t^2} = \frac{1}{9t^2}$$

Lastly, we evaluate $\frac{d^2y}{dx^2}$ when $t = -1$.

$$\frac{d^2y}{dx^2}|_{t=-1} = \frac{1}{9\cdot(-1)^2} = \frac{1}{9}$$