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1. Use the differentiation rules to find the derivative of

$$(x^3 - x)(3e^x + x^e).$$

You do not need to simplify your answer.

Solution. We apply the product rule, the sum rule and the power rule.

$$\begin{aligned} [(x^3 - x)(3e^x + x^e)]' &= (x^3 - x)'(3e^x + x^e) + (x^3 - x)(3e^x + x^e)' \\ &= ((x^3)' - x')(3e^x + x^e) + (x^3 - x)((3e^x)' + (x^e)') \\ &= (3x^2 - 1)(3e^x + x^e) + (x^3 - x)(3e^x + ex^{e-1}) \end{aligned}$$

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2. Use the definition to find the derivative of

$$f(x) = \sqrt{x}$$

No credit for using other methods.

Solution. By the definition of derivative, we have

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$