Math 106-150 **Answers to Quiz # 3** **31 Jan 08**

. 10 1. Evaluate the following limits:

$$\text{(a)} \lim_{y\to 0} \frac{\sin 5y}{2y}, \quad \text{(b)} \lim_{x\to\infty} \frac{3x^3+1}{x^2+2x^3+2}, \quad \text{(c)} \lim_{x\to 2^-} \frac{x^2-4}{x^2+x-6}, \quad \text{(d)} \lim_{x\to -3^+} \frac{x^2-4}{x^2+x-6}.$$

Solution. For (a), we have

$$\lim_{y\to 0} \frac{\sin 5y}{2y} = \lim_{y\to 0} \frac{\sin 5y}{2y}\frac{5}{5} = \lim_{y\to 0} \frac{\sin 5y}{5y}\frac{5}{2} = 1\cdot\frac{5}{2} = \frac{5}{2}.$$

For (b), we have

$$\lim_{x\to\infty} \frac{3x^3+1}{x^2+2x^3+2} = \lim_{x\to\infty} \frac{3+1/x^3}{1/x+2+2/x^3} = \frac{3}{2}.$$

For (c), we have

$$\lim_{x\to 2^-} \frac{x^2-4}{x^2+x-6} = \lim_{x\to 2^-} \frac{(x+2)(x-2)}{(x+3)(x-2)} = \lim_{x\to 2^-} \frac{x+2}{x+3} = \frac{4}{5}.$$

For (d), we have

$$\lim_{x\to -3^+} \frac{x^2-4}{x^2+x-6} = \lim_{x\to -3^+} \frac{(x+2)(x-2)}{(x+3)(x-2)} = \lim_{x\to -3^+} \frac{x+2}{x+3} = -\infty,$$

where we've used the fact that, for x close to, but more than, -3, $x+2$ is about -1 and $x+3$ is close to but more than 0.