Math 104-004 **Answers to Test # 4a** **7 Dec 07**

. 12 1. Find the following antiderivatives:

(a) $\displaystyle\int \left(3x^4 + 6x^{1/3} + \frac{5}{e^{3x}}\right)\,dx$

(b) $\displaystyle\int \left(x^2 + \frac{3}{x}\right)^2\,dx$

Solution. For part (a),

$$\begin{aligned}\int \left(3x^4 + 6x^{1/3} + \frac{5}{e^{3x}}\right)\,dx &= \int \left(3x^4 + 6x^{1/3} + 5e^{-3x}\right)\,dx \\ &= \frac{3}{5}x^5 + \frac{6}{4/3}x^{4/3} - \frac{5}{3}e^{-3x} + C\end{aligned}$$

For part (b),

$$\begin{aligned}\int \left(x^2 + \frac{3}{x}\right)^2\,dx &= \int \left(x^4 + 6x + \frac{9}{x^2}\right)\,dx \\ &= \int \left(x^4 + 6x + 9x^{-2}\right)\,dx \\ &= \frac{1}{5}x^5 + 3x^2 - 9x^{-1} + C\end{aligned}$$

. 8 2. For the demand function $q = 1000 - 50p$, find the elasticity E and the value of q at which total revenue is maximixed.

Solution. Notice that $\frac{dq}{dp} = -50$ so

$$\begin{aligned}E &= -\frac{p}{q}\frac{dp}{dq} \\ &= -\frac{p}{1000 - 50p}-50 \\ &= \frac{50p}{1000 - 50p} = \frac{p}{20 - p}\end{aligned}$$

Total revenue is maximized when $E = 1$, so we solve

$$\begin{aligned}\frac{p}{20 - p} &= 1 \\ p &= 20 - p \\ 2p &= 20\end{aligned}$$

and so $p = 10$ to maximize revenue. To find q, notice that $q = 1000 - 50p = 1000 - 500 = 500$.

. 14 3. Evaluate the following definite integrals:

(a) $\displaystyle\int_1^4 \left(x^5 + 6x + 5\right)\, dx$

(b) $\displaystyle\int_0^1 \left(e^{4x} + 3x^2\right)\, dx$

Solution. For part (a),

$$\begin{aligned}\int_1^4 \left(x^5 + 6x + 5\right)\, dx &= \frac{1}{6}x^6 + 3x^2 + 5x\Big|_1^4 \\ &= \left(\frac{2048}{3} + 48 + 20\right) - \left(\frac{1}{6} + 3 + 5\right) \\ &= \frac{1485}{2}\end{aligned}$$

For part (b),

$$\begin{aligned}\int_0^1 \left(e^{4x} + 3x^2\right)\, dx &= \frac{1}{4}e^{4x} + x^3\Big|_0^1 \\ &= \left(\frac{e^4}{4} + 1\right) - \left(\frac{e^0}{4} + 0\right) \\ &= \frac{e^4 + 3}{4} = 14.3995\end{aligned}$$

. 8 4. Given that $\displaystyle\int_1^3 g(x)\, dx = 2$, find

(a) $\displaystyle\int_1^3 (x - g(x))\, dx$

(b) $\displaystyle\int_3^1 5g(x)\, dx$

Solution. For part (a),

$$\begin{aligned}\int_1^3 (x - g(x))\,dx &= \int_1^3 x\,dx - \int_1^3 g(x)\,dx \\ &= \left.\frac{x^2}{2}\right|_1^3 - 2 \\ &= \frac{3^2 - 1}{2} - 2 = 2\end{aligned}$$

For part (b),

$$\int_3^1 5g(x)\,dx = -\int_1^3 5g(x)\,dx = -5\int_1^3 g(x)\,dx = -5 \cdot 2 = -10.$$

. 16 5. Use a substitution to find the following antiderivatives:

(a) $\displaystyle\int 8xe^{2x^2}\,dx$

(b) $\displaystyle\int \left(\frac{\ln z}{z}\right)\,dz$

Solution. For part (a), we use the substitution $u = 2x^2$, so $du = 4x\,dx$ and $2\,du = 8x\,dx$. Thus, we have

$$\int e^u(2du) = 2\int e^u\,du = 2e^u + C = 2e^{2x^2} + C$$

For part (a), we use the substitution $u = \ln z$, so $du = dx/x$ Thus, we have

$$\int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln z| + C$$

. 8 6. Use a substitution to evaluate $\displaystyle\int_2^4 \frac{x^2}{3 + 2x^3}\,dx$.

Solution. We use the substitution $u = 3 + 2x^3$, so $du = 6x^2\,dx$ and $du/6 = x^2\,dx$. Also, if $x = 2$, then $u = 19$ and if $x = 4$, then $u = 131$. Thus, we have

$$\int_{19}^{131} \frac{1}{u}\frac{du}{6} = \frac{1}{6}\ln|u|\Big|_{19}^{131} = \frac{\ln(131) - \ln(19)}{6} = 0.3218$$

. 8 7. Find the cost function $C(x)$ if the **marginal** cost function is $C'(x) = x^2 + 4x + 2$ and $C(2) = 2/3$.

Solution. Notice that $C(x) = x^3/3 + 2x^2 + 2x + K$, so

$$\frac{2}{3} = C(2) = \frac{8}{3} + 8 + 4 + K.$$

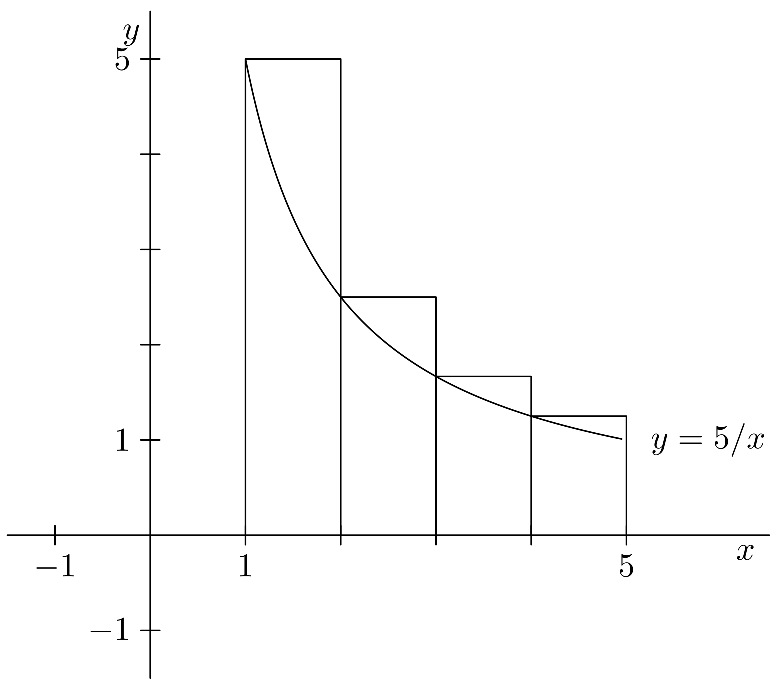
Solving, $K = -14\frac{2}{3} + \frac{2}{3} = -14$ and so $C(x) = x^3/3 + 2x^2 + 2x - 14$.

. 12 8. Using rectangles, approximate the area under the graph of $y = 5/x$ on the interval $[1, 5]$ using $n = 4$ and **left** endpoints. Mark the four rectangles on the axes below.

Solution. Notice that the four rectangles have bases $[0, 1]$, $[1, 2]$, $[2, 3]$ and $[3, 4]$. So the heights are, respectively $f(0) = 4$, $f(1) = (16 - 1)/4 = 15/4$, $f(2) = (16 - 4)/4 = 3$, and $f(3) = (16 - 9)/4 = 7/4$. The total area is

$$f(0) \cdot 1 + f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 = 4 + \frac{15}{4} + 3 + \frac{7}{4} = 12.5$$

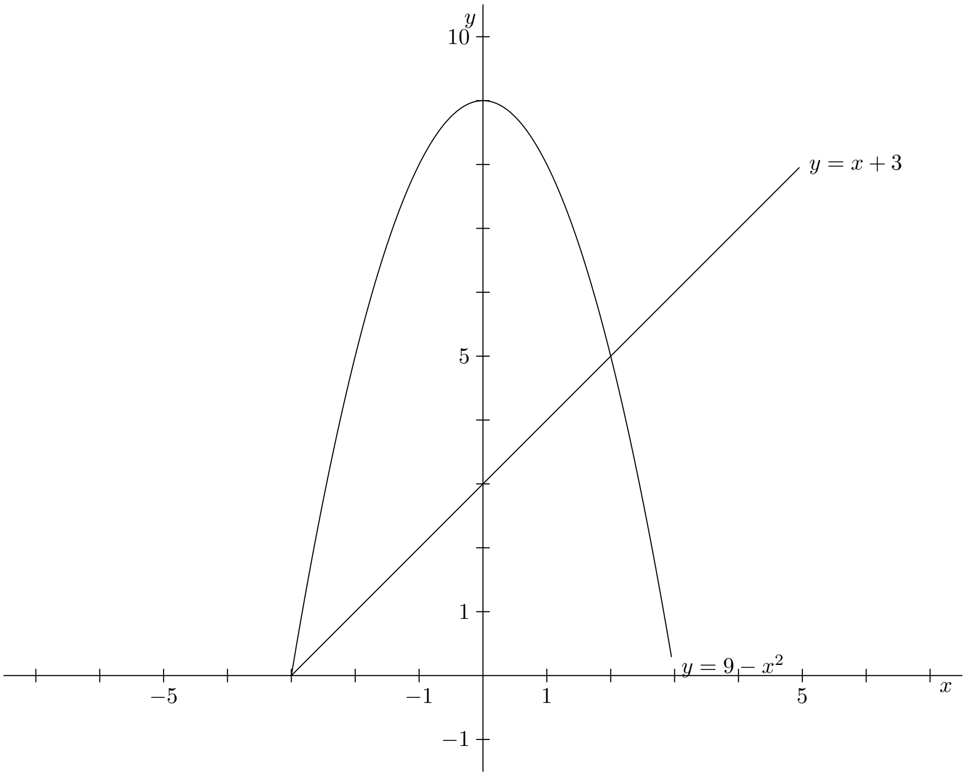
The rectangles are



. 14 9. Let R be the region bounded by $y = 9 - x^2$ and $y = x + 3$.

(a) Sketch R, including its intersection points, on the axes below.

(b) Find the area of R.



Solution. The graphs are:
To find the intersection points, solve $9-x^2 = x+3$, which gives $x^2+x-6 = (x-2)(x+3)$ so $x = 2$ or $x = -3$. When $x = 2$, we have $y = 9-2^2 = 5$ and when $x = -3$, $y = 9-(-3)^2 = 0$. So the intersection points are $(2,5)$ and $(-3,0)$.
To find out which curve is the upper one, pick 0 (any number between -3 and 2 will do) and substitute into each curve: for $y = 9-x^2$, we have $9-0$ and for $y = x+3$, we have 3. Thus, $y = 9-x^2$ is the 'bigger' function and the integral is

$$\begin{aligned}\int_{-3}^{2} (9-x^2)-(x+3)\,dx &= \int_{-3}^{2} 6-x-x^2\,dx \\ &= 6x - \frac{x^2}{2} - \frac{x^3}{3}\bigg|_{-3}^{2} \\ &= \left(12-2-\frac{8}{3}\right) - \left(-18-\frac{9}{2}+9\right) \\ &= \frac{125}{6} = 20.83\end{aligned}$$