. 24 1. Find $f'(x)$ for the following functions. (You need not simplify your answers.)

(a) $f(x) = \dfrac{(2x+3)^3}{4x^2+1}$

(b) $f(x) = e^{-2x^2+x} + \ln(x^2 + 2e^{-3x})$

(c) $f(x) = (x^2-1)^4 e^{3x^2}$

Solution. For part (a), we use the quotient rule to get

$$f'(x) = \frac{(4x^2+1)3(2x+3)^2 \cdot 2 - (2x+3)^3 8x}{(4x^2+1)^2}.$$

For part (b), we use the rules for $e^{h(x)}$ and $\ln h(x)$ to get

$$f'(x) = e^{-2x^2+x}(-4x+1) + \frac{2x - 6xe^{-3x}}{x^2+2e^{-3x}}.$$

For part (c), we use the product rule and then the chain rule to get

$$f(x) = 4(x^2-1)^3 \cdot 2x \cdot e^{3x^2} + (x^2-1)^4 \cdot e^{3x^2} \cdot 6x.$$

. 12 2. Solve the following equations for x:

(a) $16^{-x+1} = 8^{2x}$

(b) $2e^{3x+4} = 10$

Solution. For part (a), we have

$$\begin{aligned}(2^4)^{-x+1} &= (2^3)^{2x} \\ 2^{4(-x+1)} &= 2^{6x} \\ 4(-x+1) &= 6x \\ -4x+4 &= 6x \\ 4 &= 10x,\end{aligned}$$

and so $x = 4/10$.

For part (b), we have

$$\begin{aligned} e^{3x+4} &= \frac{10}{2} = 5 \\ \ln(e^{3x+4}) &= \ln 5 \\ 3x + 4 &= \ln 5 \\ 3x &= (\ln 5) - 4 \\ x &= \frac{(\ln 5) - 4}{3} \end{aligned}$$

. 8 3. I need to have \$60,000 in 18 years to pay my son's college tuition. How much must I invest now in an account paying 10% compounded annually to have the required amound in 18 years?

Solution. Using the equation

$$P = A\left(1 + \frac{r}{m}\right)^{mt}$$

with $P = 60000$, $r = .10$, $m = 1$, and $t = 18$, we have $60,000 = A(1.1)^{18}$ and so $A = 60,000/(1.1)^{18}$, which gives \$10,791.53 as the required amount to invest now.

. 10 4. Find the equation of the tangent line to the graph of the curve $y = f(x) = (5x^2 - 3)^3$ at $x = 1$.

Solution. First, we find $f(1) = (5 \cdot 1^2 - 3)^3 = 2^3 = 8$ and then $f'(x) = 3(5x^2 - 3)^2(10x)$ so $f'(1) = 3(5 \cdot 1^2 - 3)^2 \cdot 10 = 3 \cdot 4 \cdot 10 = 120$. Thus, the equation of the tangent line is

$$y - 8 = 120(x - 1).$$

. 10 5. Cesium-134 has a half-life of 2.065 years. How much of a 60 gram mass is left after 10 years?

Solution. Let $C(t)$ be the amount of Cesium, in grams, after t years. Then we know that $C(0) = 60$, $C(2.065) = 30$ and we have to find $C(10)$.
The general formula for $C(t)$ is Ae^{kt} and to find A we notice that $60 =$

$C(0) = Ae^{k \cdot 0} = A$. To find k, we solve

$$
\begin{aligned}
30 &= C(2.065) = 60e^{k \cdot 2.065} \\
\frac{1}{2} &= e^{k \cdot 2.065} \\
\ln\left(\frac{1}{2}\right) &= k \cdot 2.065 \\
k = \frac{\ln\left(\frac{1}{2}\right)}{2.065} &= -.3357
\end{aligned}
$$

Thus, $C(10) = 60e^{-.3357 \cdot 10} = 2.09$ grams.

18 6. Let $f(x) = \frac{1}{2}x^4 - 4x^3 + 9x^2 + 3$, for all numbers x.

(a) Find all critical numbers of $f(x)$.

(b) Chart $f'(x)$ on a number line.

(c) List the open intervals on which $f(x)$ is an increasing function.

(d) List the open intervals on which $f(x)$ is an decreasing function.

Solution. Notice that

$$
\begin{aligned}
f'(x) &= 2x^3 - 12x^2 + 18x \\
&= 2x(x^2 - 6x + 9) \\
&= 2x(x-3)^2
\end{aligned}
$$

Since $f'(x)$ is never undefined, the only critical numbers are where $f'(x) = 0$, that is, 0 and 3.

Since $f(x)$ is never undefined, the only points where we have to split the number line are 0 and 3. For the interval $(-\infty, 0)$, we pick $x = -1$ and get $f'(-1) = 2(-1)(-4)^2 < 0$. For the interval $(0, 3)$, we pick $x = 1$ and get $f'(1) = 2 \cdot 1(-3)^2 > 0$. For the interval $(3, +\infty)$, we pick $x = 4$ and get $f'(1) = 2 \cdot 4 \cdot 1^2 > 0$.

Thus, $f(x)$ is increasing on $(0, 3)$ and $(3, +\infty)$ and it is decreasing on $(-\infty, 0)$.

12 7. How long does it take (in years) for \$20,000 to double in value, if it is invested in an account paying 6% compounded monthly?

Solution. Using the equation

$$P = A\left(1 + \frac{r}{m}\right)^{mt}$$

with $A = 20000$, $P = 40000$, $r = .06$, and $m = 12$, we have

$$\begin{aligned} 40000 &= 20000(1.005)^{12t} \\ 2 &= (1.005)^{12t} \\ \ln 2 &= \ln(1.005)^{12t} \\ \ln 2 &= 12t \ln 1.005 \\ t &= \frac{\ln 2}{12 \ln 1.005} = 11.581 \end{aligned}$$

Thus, it will take 11.581 years or, to be precise, 11 years and 7 months.

6 8. If $\log_b 2 = a$ and $\log_b 5 = c$, express $\log_b 100$ in terms of a and c?

Solution. Since $100 = 2^2 \cdot 5^2$, by the properties of logarithms,

$$\begin{aligned} \log_b 100 &= \log_b(2^2 \cdot 5^2) \\ &= \log_b 2^2 + \log_b 5^2 \\ &= 2 \log_b 2 + 2 \log_b 5 = 2a + 2c. \end{aligned}$$