Control Theory Techniques Applied to Biological Population Problems

Project Mentors: Professor Richard Rebarber (Department of Mathematics), Professor Andrew Tyre (School of Natural Resources).

Prerequisites: Introductory course in matrix theory. Some introductory mathematical analysis and complex analysis would also be helpful, but are not strictly required. Familiarity with Matlab, Maple or Mathematica would also be helpful, but are not required.

Project Description: In this project we will apply techniques from the subject of *mathematical control theory* to problems in population dynamics. When analyzing how the population of an animal or a plant changes with time, the transition rates of survival, growth and reproduction can be incorporated into a *population projection matrix*. The eigenvalues of this matrix provide predictions for the behavior of the population. For instance, if the largest eigenvalue modulus, called the spectral radius, is less than 1, then the population decreases no matter what the initial conditions are. An important task in modern ecology is to predict what happens to the population dynamics when these transition rates are perturbed (that is, changed); the perturbations can be described by one or more parameters. This could happen due to uncertainty in the model, or these rates could be intentionally changed in order to get a desired result, such as maintaining an endangered species or eradicating a predator.

The relationship between the perturbed eigenvalues and the original eigenvalues is very nonlinear. This problem is typically analyzed by biologists using the calculus technique of local linearization, but this approach often isn't accurate. There are many techniques used in control theory for the detailed analysis of perturbed eigenvalues, especially when the structure of the perturbation is known (which is typically the case in biological systems). In the simplest situation when there is one parameter, there are techniques to analyze the precise relationship between the perturbed eigenvalues and the parameter. For multiple parameters, there are techniques from the area of *robust control* which can determine the parameter size required to obtain desired results. Furthermore, *pseudospectra* and *spectral value sets* can be used to analyze the effects of modelling uncertaintly.

In this project, we will study: how to derive population projection matricies; how to model perturbations to the matrix; how to describe population problems mathematically; traditional methods which have been used for such problems; relevant robust control techniques; and the application of these techniques to biological problems.

There are many biological systems which are amenable to these techniques. We briefly describe here one such population model. A population projection matrix has been developed for southeastern U.S. loggerhead sea turtles, which is an endanged species. The matrix contains entries which describe the survival and reproductive rates between the turtle' stages of development. The population can be manipulatived beneficially by protection of nests or removal of egss to protected hatcheries, which can be described by parameters in the matrix. The populations is also negatively impacted by the incidental capture and drowning in shrimp trawls. These deaths can be decreased by the use of "turtle excluder devices" (TEDs), which are escape hatches inserted into existing shrimp nets. The number of TEDs is another parameter in the matrix. In this project we will learn how to use control theory techniques to choose the parameters to obtain population growth, and to analyze how our choices are affected by model uncertainty.