

Nontext Problem 2

Due: Monday, April 25

A basic model in mathematical biology for certain animal population $P(t)$ as a function of time is the famous Malthus equation of exponential population

$$\begin{aligned}\frac{dP}{dt} &= rP \\ P(0) &= P_0.\end{aligned}$$

Here r is a growth rate. We learned early on in Calculus that the solution to this problem is

$$P(t) = P_0 e^{rt}.$$

Of course, it was long ago realized that this simple model is inadequate for correct descriptions of populations either qualitatively or quantitatively. The reason is that there is a limit to growth imposed by the ambient environment. Subsequently, a correction was proposed, namely the logistic model

$$\begin{aligned}\frac{dP}{dt} &= rP \left(1 - \frac{P}{K}\right) \\ P(0) &= P_0.\end{aligned}$$

Here K is a limiting population size and r a growth rate. The solution to this problem is also a calculus level problem and it can be stated in the form

$$P(t) = \frac{P_0 K e^{rt}}{K - P_0 + P_0 e^{rt}}.$$

Generally, the logistic model isn't too good for predicting human populations, but fares well with animal populations. There are exceptions, of course, and we are going to focus on one such. Following is a table of the populations of England and Wales from 1700 to 2001. This data is listed in the file `myparms.m` which, when run from Matlab or Octave, will declare a global variable `myP` and fill it with rows whose entries are (year-1700, population in millions).

Here is the problem of the exercise: find a logistic type function that best fits this data. This is not a trivial problem. I've added some capability to `Optimize` that will help in generic parameter fitting problems. Basically, you have to program a logistic residual and a logistic gradient with respect to the parameters as a row vector. Then use the `leastsq*` functions. See `Optimize` for some material. We'll illustrate it in class with our favorite, the Rosenbrock.

Your assignment is simply to play with this problem and report on your results.