Course Assignments for Math 496, Section 006

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Note: Unless otherwise stated, it is always permissible to use Matlab for calculations. As a general rule, you are expected to show your work. In particular, if you use Matlab to solve a problem, you are expected to provide a transcript via diary or cut and paste into a document. Typed documents are preferred; hand written copy will be accepted if it is *neatly* written. Unless otherwise stated, hardcopy is the rule.

Assignment 1

Points: 25
Due: January 24

1. Exercise 1.1 (p. 14) of text. (Here is a hint that you may find helpful for bookkeeping: given an $m \times n$ matrix $A = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n]$, where each $\mathbf{a}_j$ is a column of A, and given a column vector $\mathbf{x} = [x_1, x_2, \ldots, x_n]^T$ then we may write$A\mathbf{x} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n$.)

2. Use 5 data points (your choice) to come up with the very best fitting of a quadratic $y = x_1 + x_2 t + x_3 t^2$ to the curve $y = \sin(\pi t)$, $0 \leq t \leq 1$.

3. Exercise A.15 of text (p.278). This will help familiarize you with an important, slightly nonstandard, vector norm.

4. Create a multiple plot consisting of four subplots arranged in a square, where the subplots are graphs of $y = \sin(\pi t)$, $0 \leq t \leq 1$ together with an approximation to the sin function, namely, (i) $y = \pi t$, (ii) $y = \pi t - (\pi t)^3/6$, (iii) $y = 4t(1-t)$and (iv) your approximation from Exercise 2.

Assignment Finalized

Assignment 2

Points: 30
Due: February 7

1. Textbook Exercise B4 of Appendix B. (Hint: look at Example B.6 of the text.)

2. Textbook Exercise B8 of Appendix B.

3. Textbook Exercise B10 of Appendix B.
   About the probability plot: the function `hist` accepts input data returns a vector of bin values and does a histogram plot. Of course the area under the bars is not 1. In contrast, a probability plot uses the data to approximate a p.d.f. with a bar graph, so the area under the bars should be approximately 1. Hence, once you have your data in, say, the row vector `data`, do something like
   ```
   hist(data)
   ```
   to get an idea of good bin sizes and range, e.g., you might settle on bin widths of 0.3, so that you would set
   ```
   databin = -2.5:0.3:2.5;
   hst = hist(data,databin);
   ```
   Now you need to scale the histogram values, then do a plot. But do not use hist again. Rather, use the `bar` plotting function with appropriate data bin, i.e., something like
   ```
   scaledhst = hst/0.3/sum(hst);
   bar(databin,scaledhst);
   ```

Assignment Finalized

Assignment 3

Points: 35
Due: February 21

1. Textbook Exercise 3.1.
2. Textbook Exercise 3.3.
3. Textbook Exercise 3.4.
4. Textbook Exercise 2.1 (a), (d), (g) and also: (j) Find 95% confidence intervals for the parameters of the problem. (For (g) you will find in text CD a `irls.m` routine from Example 4 of Chapter 2.)
5. Textbook Exercise 4.1.

Assignment Finalized

Assignment 4

Points: 35
Due: March 7

1. Use the definitions of Moore-Penrose inverse and matrix norm to show that $\left\|G^{\dagger}\right\|_2 = 1/\sigma_p$, where $\mathrm{rank}\,(G) = p$ and σ_p is the p-th singular value of G.
2. Exercise 4.3 (the files you need are on your textbook CD in the directory /Homework/chap4/prob2.)

Assignment Finalized

Assignment 5

Points: 45
Due: April 7

Note: Before you start coding up these exercises, take a look at the functions that are available to you in the directory MatlabTools/regutools. Lots of work is already done for you.

1. Textbook Exercise 5.2
2. Textbook Exercise 5.3
3. Textbook Exercise 5.5
4. Verify by using the approach and calculations done in class that with higher order Tikhonov regularization the resolution matrix is given by $R_{m,\alpha,L} = XFX^{-1}$, where F is the diagonal matrix of filter factors.

Assignment Finalized

Assignment 6

Points: 40
Due: April 26

1. Textbook Exercise 7.1.
2. Following Example 7.2 of the text, we consider the Shaw problem, where the model is constructed as in the `examp.m` file, except that the spikes are defined by
   ```
   xtrue(6) = 1000;
   xtrue(15) = 50;
   xtrue(16) = 50;
   xtrue(19) = 500;
   ```
   Use nonnegative least squares, zeroth order Tikhonov regularization, maximum entropy and total variation to solve this problem and graph your results, along with the true model. Use the same random perturbations as in the CD file.
3. Textbook Exercise 9.2.
4. Textbook Exercise 10.1

Assignment Finalized

Assignment 7

Points: 20
Due: April 26

I am going to pro-rate your prior assignment score to account for the last 20 points.

Assignment Finalized