

Instructions: Point values of problems are given in parentheses. This exam is worth 130 points. Note the following:

- Papers must be written neatly and legibly. Your solutions must be readable, i.e., consist of narratives with sentences and connectives between equations.
- Show your work. Solutions should be complete. Do not expect me to run any .m files to obtain your answers, although you must include such files or diaries as evidence of your work. Get Matlab help on “diary” if you’re still unclear on this. When finished, you can edit your diary file with any text editor. Eliminate unnecessary output.
- All work is to be done by individuals, with no collaboration or consultation of any kind with anyone else other than me. If you use any outside sources other than the text or notes, they should be clearly cited. If you have questions about any problems, contact me by email ASAP. I will post my comments on our Blackboard discussion board if this is appropriate.
- Exam files must handed in at class time on Thursday, October 22.

(15) **1.** What point of the plane $3x + 2y + z - 6 = 0$ in \mathbb{R}^3 is closest to the origin when measured by the (a) 1-norm, (b) 2-norm, and (c) ∞ -norm?

(24) **2.** Determine the best approximation $p(x)$ to $f(x) = x^3$ from the subspace \mathcal{P}_1 of $C[-1, 1]$. Then compute the 1-norm, 2-norm and ∞ -norm of the error function $e(x) = f(x) - p(x)$.

(28) **3.** Use a divided difference table to derive a formula (as we did in class for Hermite cubics) for a quadric polynomial that interpolates $f(a)$, $f'(a)$, $f''(a)$, $f(b)$ and $f'(b)$ for a given $f \in C[a, b]$. Code this up as a Matlab function with appropriate inputs and test on $f(x) = (x + 1)^4 \in C[0, 1]$.

(28) **4.** Suppose we want to approximate $f(x) = \sin x$ for all x .

(a) Use the Matlab file PPfcn.m to compute a Hermite p.p. approximation to the function of (a) with knots at $0, \pi/4, \pi/2$ and compute the infinity norm of the error.

(b) Compare this error the the error of a best minimax polynomial (use minimaxd.m) whose evaluation requires the same (or nearly so) number of flops as in (a). Who wins?

(20) **5.** Let \mathcal{A} be the set of multiples of the function $g(x) = x$, $0 \leq x \leq 1$, and the operator $X : C[0, 1] \rightarrow \mathcal{A}$ be given by

$$(Xf)(x) = 2 \int_0^1 xf(y)dy.$$

where $f \in C[0, 1]$, a normed space with the infinity norm. Show that X is a linear projection operator (see pp. 22-23 of text) and estimate its norm.

(15) **6.** Use any of the key facts from Chapter 7 to explain why the levelled reference error is guaranteed to increase from step to step if the current reference is not optimal.