

TAKE HOME FINAL FOR MATH 441, FALL 2009

TakeHome Final Exam

CS/Math 441

Fall 2009

Name: \_\_\_\_\_

Score: \_\_\_\_\_

*Instructions:* This exam is due on Wednesday, December 16, by 5:00 pm. Email me if you have any questions. I will respond individually or to everyone if appropriate.

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(40) **1.** Let  $f(x) = 1$  and  $g(x) = 1 - 2x$  be viewed as elements of the vector space  $C[0, 1]$ .

(a) Calculate  $\|f\|_p$ , where  $p = 1, 2, \infty$ .

(b) Show that  $f$  and  $g$  are orthogonal with respect to the inner product  $\langle f, g \rangle = \int_0^1 f(x)g(x)dx$ .

(c) Find the best least squares approximation to  $h(x) = x^3$  from the subspace spanned by  $f(x)$  and  $g(x)$  by using the normal equations. (Note: this subspace is just  $\mathcal{P}_1$ , where the polynomials are viewed as functions on  $[0, 1]$ .)

(20) **2.** Do the following calculations by hand. You are given that  $f(0) = 1$ ,  $f(1) = 0$ ,  $f(2) = 1$  and  $f(4) = 2$ .

Find a polynomial of least degree that interpolates this function at these points and use it to estimate  $f(3)$  and  $f(6)$ . Which value are you more likely to believe and why? (Hint: look at the form of the error estimate of Equation 4.13 on page 35 of your text.)

(80) **3.** Let  $f(x) = x \sin x$ ,  $0 \leq x \leq 2\pi$ . Use Matlab to graph the error of approximation in four of the following cases:

(a) The best minimax approximation to  $f(x)$  from  $\mathcal{P}_8$ .

(b) A complex trigonometric polynomial of degree  $N = 8$ .

(c) A Hermite cubic piecewise polynomial with 6 equally spaced knots.

(d) A clamped cubic spline with 6 equally spaced knots.

(e) A natural cubic spline with 6 equally spaced knots.

The tools available to you are all of our Matlab functions in Course Materials, including those in a folder named PPFcns, where all of the piecewise tools are located. Consult our ClassroomNotes file for helpful scripts, more information on concepts. There is a complete discussion of splines in the last section.