Exam 3 **Math 314** **Fall 2006**

Name:______________________________ **Score:**__________

Instructions: Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., π) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

(25) **1.** Let $\mathbf{v}_1 = (1, 1, 0))$, $\mathbf{v}_2 = (-1, 1, 1)$, $\mathbf{v_3} = (1/2, -1/2, 1)$ and $\mathbf{v} = (1, 2, -2)$.

(a) Find the norm of $\mathbf{v}$.

(b) Find the cosine of the angle between the vectors $\mathbf{v}$ and $\mathbf{v}_1$.

(c) Verify the CBS inequality for the pair of vectors $\mathbf{v}, \mathbf{v}_2$.

(d) Show $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v_3}$ is an orthogonal set (hence a basis of $\mathbb{R}^3$).

(e) Find the coordinates of $\mathbf{v}$ relative to this basis.

(10) **2.** Set up and solve the normal equations for the system $A\mathbf{x} = \mathbf{b}$, where$A = [\mathbf{v}_1, \mathbf{v}_2]$, $\mathbf{v}_1 = (1, 1, 0))$, $\mathbf{v}_2 = (-1, 1, 1)$ and $\mathbf{b} = (2, 1, 1)$. Is the least squares solution a genuine solution?

3. (14) Find an eigensystem for the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Give reasons why the matrix A is or is not diagonalizable.

(24) **4.** Matrix $A = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$ has eigenvalues -3, 2, and eigenvectors $\mathbf{v}_1 = (2, -1)$, $\mathbf{v}_2 = (1, 2)$.

(a) Use this information to find a diagonalizing matrix P for A and resulting diagonal matrix D.

(b) Use (a) to find a formula for powers of A in terms of powers of eigenvalues of A.

(c) Find unit vectors $\mathbf{u}_1$, $\mathbf{u}_2$ in the directions of $\mathbf{v}_1$, $\mathbf{v}_2$, respectively, and exhibit an orthogonal matrix U that diagonalizes A.

(18) **5.** Fill in the blanks, or answer T/F:
(a) If A is a real matrix, then A^TA is symmetric (T/F) ____________.

(a) Eigenvalues of a matrix cannot be zero (T/F) ____________.

(b). If Q is an $n \times n$ orthogonal matrix and $\mathbf{v} \in \mathbb{R}^n$, then $\|Qv\| = \|v\|$ (T/F)____________.

(c) If $\rho(A) < 1$ and $\mathbf{x}^{(k+1)} = A\mathbf{x}^{(k)}$, then $\lim_{k\to\infty} \mathbf{x}^{(k)}$ equals ____________________.

(d) The matrix $\frac{1}{\sqrt{3}} \begin{bmatrix} 1+i & i \\ i & 1-i \end{bmatrix}$ is not unitary (T/F) ____________________ .

(e) The matrix $\begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is diagonalizable because ______________________________ .

(f) The matrix $\begin{bmatrix} 1 & i \\ -i & 3 \end{bmatrix}$ is unitarily diagonalizable because ________________________ .

(g) Every orthonormal set of vectors is linearly independent (T/F) ________.

(h) The component of $\mathbf{u} = (1, 2, 0)$ along the vector $\mathbf{v} = (1, 1, 1)$ is $\text{comp}_{\mathbf{v}}(\mathbf{u}) =$____________.

(9) **6.** The vector $\mathbf{v}_1 = (1, 1)$ is an eigenvector for the symmetric matrix $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$.

(a) Find a vector orthogonal to $\mathbf{v}_1$ and show it is an eigenvector of A.

(b) (**Honors students only**) For real eigenvectors $\mathbf{v}_1$, $\mathbf{v}_2$ of a real symmetric matrix A corresponding to (real) eigenvalues $\lambda_1 \neq \lambda_2$, we have $\mathbf{v}_1^T A \mathbf{v}_2 = \mathbf{v}_2^T A \mathbf{v}_1$. Use this to deduce that $\mathbf{v}_1^T \mathbf{v}_2 = 0$.