**Exam 2** **Math 314** **Fall 2006**

**Name:**______________________________ **Score:**__________

*Instructions:* Show your work in the spaces provided below for full credit. Use the reverse side for additional space, *but clearly so indicate.* You must clearly identify answers and show supporting work to receive any credit. Exact answers (e.g., $\pi$) are preferred to inexact (e.g., 3.14). Point values of problems are given in parentheses. Notes or text in *any* form not allowed. Calculator is allowed.

---

(30) **1.** Let $A = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4] = \begin{bmatrix} 1 & 4 & 1 & -1 \\ 2 & 4 & 1 & -1 \\ 4 & 8 & 2 & -2 \end{bmatrix}$ with reduced row echelon form $R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Find a basis for $\mathcal{R}(A)$, the row space of $A$.

(b) Find a basis for $\mathcal{C}(A)$, the column space of $A$.

(c) Find a basis for $\mathcal{N}(A)$, the null space of $A$.

(d) Find all possible linear combinations of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ that sum to $\mathbf{0}$.

(e) Which $\mathbf{v}_j$'s are redundant in the list of vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$?

(f) Find a basis of $\mathbb{R}^3$ containing a basis of $\mathcal{C}(A)$.

(16) **2.** Use the Subspace Test to decide if $W$ is a subspace of the vector space $V$, where
(a) $V = \mathbb{R}^3$ and $W = \{(a, b, a - b + 1) \mid a, b \in \mathbb{R}\}$

(b) $V = C[0,1]$, the continuous functions on $[0,1]$ and $W = \{f(x) \mid f(x) \in C[0,1] \text{ and } f(1) = 0\}$.

(10) **3.** Assume that $1 + x$, $x + x^2$, $1 - x$ is a basis of $\mathcal{P}_2$, the space of polynomials of degree at most two, and find the coordinates of $2 + x^2$ relative to this basis.

(8) **4.** Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$. Find the adjoint matrix $\operatorname{adj}(A)$ of $A$.

(12) **5.** You are given that $\mathbf{w}_1 = (0,1,0)$, $\mathbf{w}_2 = (1,1,1)$ is a linearly independent set in $V = \mathbb{R}^3$ and $\mathbf{v}_1 = (1,3,1)$, $\mathbf{v}_2 = (2,-1,1)$, $\mathbf{v}_3 = (1,0,1)$ is a basis of $V$. Steinitz substitution says that $\mathbf{w}_1$, $\mathbf{w}_2$ can be substituted into the basis in place of certain $\mathbf{v}_i$'s. Which substitutions work?

(16) **6.** Fill in the blanks or answer True/False (T/F).

(a) Every vector space is finite dimensional (T/F) ____________.

(b) Elementary row operations on a matrix do not change the column space (T/F) ____________.

(c) If $\mathbf{x} = \mathbf{x}_0$ and $\mathbf{x} = \mathbf{x}_1$ are both vector solutions to the linear system $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x}_1 - \mathbf{x}_0$ is in the null space of $A$. (T/F) ____________.

(d) The function $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T((x, y)) = (x + y, x - 2y)$ is linear (T/F) ____________ and one-to-one (T/F) ____________.

(e) The Basis Theorem asserts that every finite dimensional vector space ________________________.

(f) The Dimension Theorem asserts that ____________________________________.

(g) A basis of a vector space is by definition____________________________________.

(10) **7.** (a) Show that the columns of the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$ form a linearly dependent set.

(b) (*Honors students only*) Prove that any set of vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n$ in a vector space $V$ that contains the zero vector is a linearly dependent set.